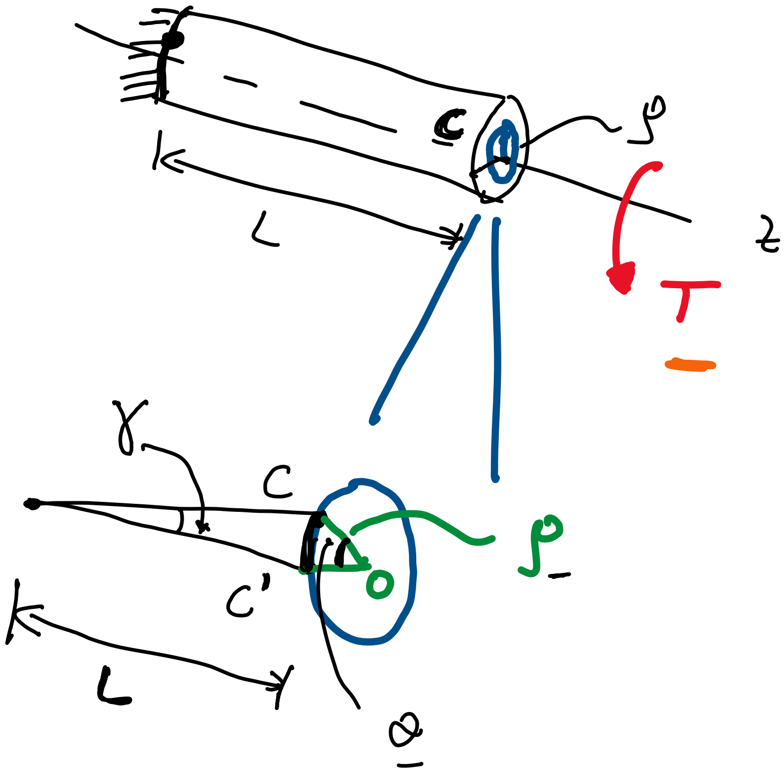
Torsion



$$cc' = L\gamma = \rho\theta$$

$$\boxed{\gamma = \frac{\rho\theta}{L} \qquad \gamma_{max} = \frac{c\theta}{L}}$$

γ = Shear strain

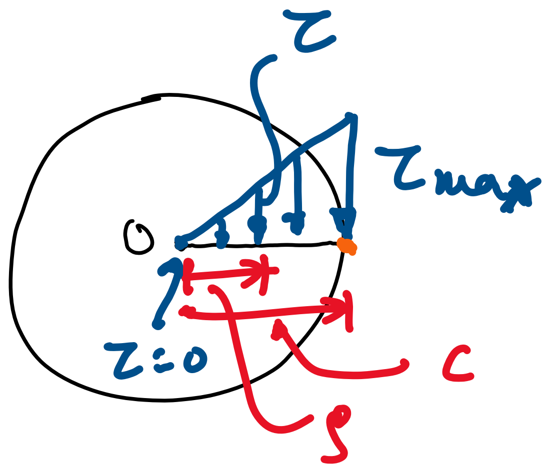
θ = angle of twist

ρ, c = radius

G = Bulk modulus.

$$\boxed{\tau = G\gamma = \frac{G\rho\theta}{L}}$$

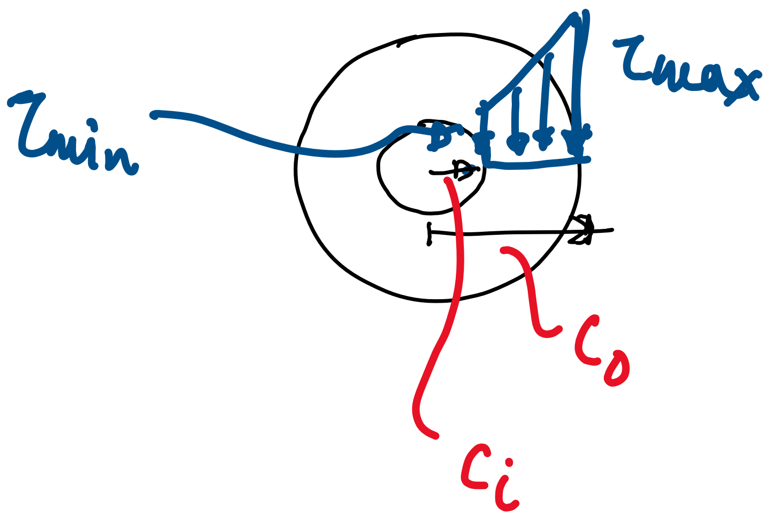
$$\boxed{\tau_{max} = G\,\gamma_{max} = \frac{G\,c\,\theta}{L}}$$



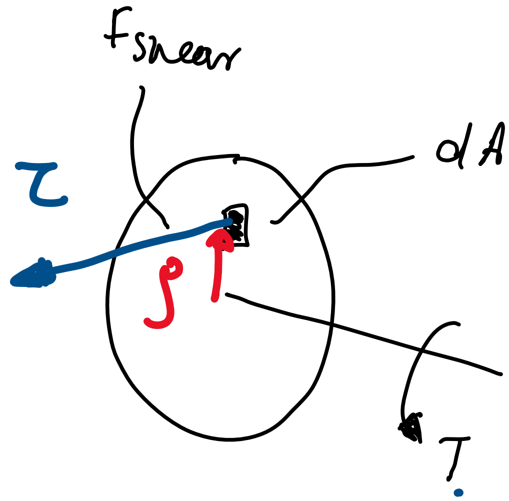
$$\tau = G\gamma = \frac{G\rho\theta}{L}$$

$$\tau_{max} = G\gamma_{max} = \frac{Gc\theta}{L}$$

$$\boxed{\frac{\tau}{\tau_{max}} = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}}$$



$$\boxed{\frac{\tau_{min}}{\tau_{max}} = \frac{c_i}{c_o}}$$



$$dT = F_{shear}\,\rho$$

$$dT = (\tau\, dA)\,\rho$$

$$T = \int_0^T dT = \int (\tau\, dA)\,\rho \qquad — (1)$$

$$\frac{\tau}{\tau_{max}} = \frac{\rho}{c} \Rightarrow \tau = \frac{\rho}{c}\,\tau_{max} \qquad — (2)$$

Substitute (2) in (1)

$$T = \int \frac{\rho^2}{c}\,\tau_{max}\, dA$$

$$T = \frac{\tau_{max}}{c} \underbrace{\int \rho^2\, dA}_{J \,=\, \text{polar moment of inertia}}$$

A-18

$$\boxed{T = \frac{\tau_{max} J}{c}} \Rightarrow \boxed{\tau_{max} = \frac{Tc}{J}}$$

c

J

Summary

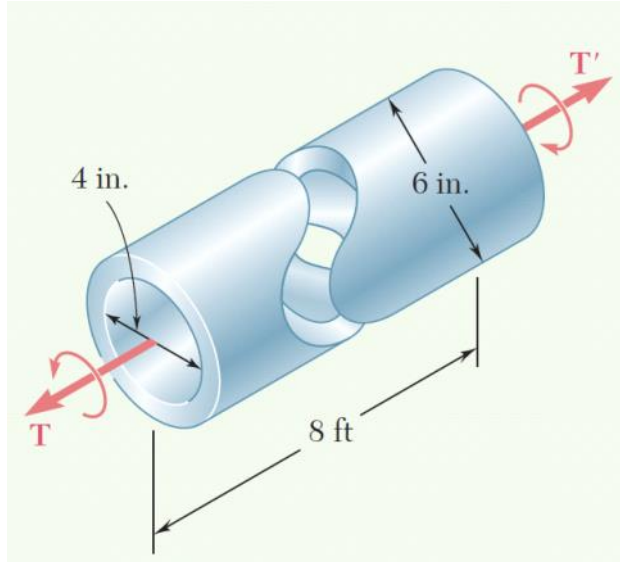
$$\gamma = \frac{\rho\theta}{L} \Rightarrow \gamma_{max} = \frac{c\theta}{L}$$

$$\tau = \frac{T\rho}{J} \Rightarrow \tau_{max} = \frac{Tc}{J}$$

$$\theta = \frac{TL}{JG}$$

Q1

The shaft shown below has an inner and outer diameters of 4in and 6in respectively. The maximum allowable shear stress is 12 ksi. Find the maximum torque that can be transmitted by the shaft without failing

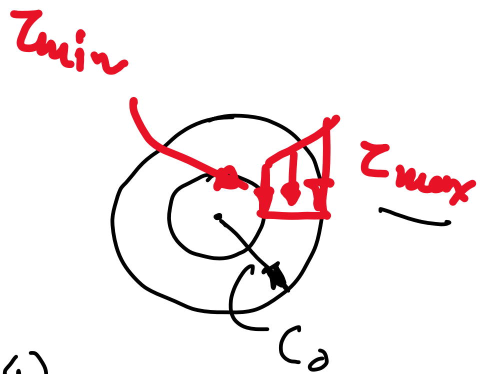


✓ $\tau_{max} = 12$ ksi;

$d_i = 4$ in; $c_i = 2$ in

$d_o = 6$ in; $c_o = 3$ in

✓ $T_{max} = ?$



$$\tau_{max} = \frac{T_{max}\, c_o}{J}$$

$J \sim \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$ ~ Table A-18

$$12 = \frac{T_{max}\ 3}{\frac{\pi}{32}\left(6^4 - 4^4\right)} \Rightarrow \boxed{T_{max} = 408.4 \text{ kip} \cdot \text{in}}$$

$\frac{\pi}{32}\left(6^4 - 4^4\right) = 102.1 \text{ in}^4$

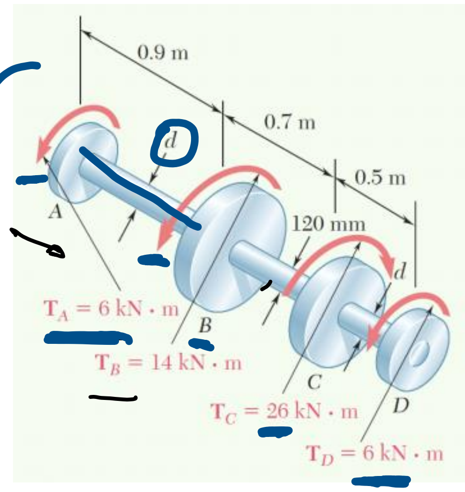
Q2

Shaft AB and CD are solid of diameter d while shaft BC is a hollow of inner and outer diameters of 90 mm and 120 mm. The shaft assembly is subjected to torques as shown. Compute

(1) Plot the Torque as a function of shaft length

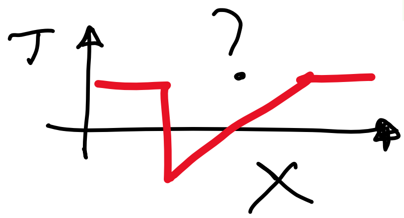
(2) Maximum and minimum shearing stress in shaft BC

(3) The diameter d if the allowable shearing stresses in shafts AB and CD are 65 MPa

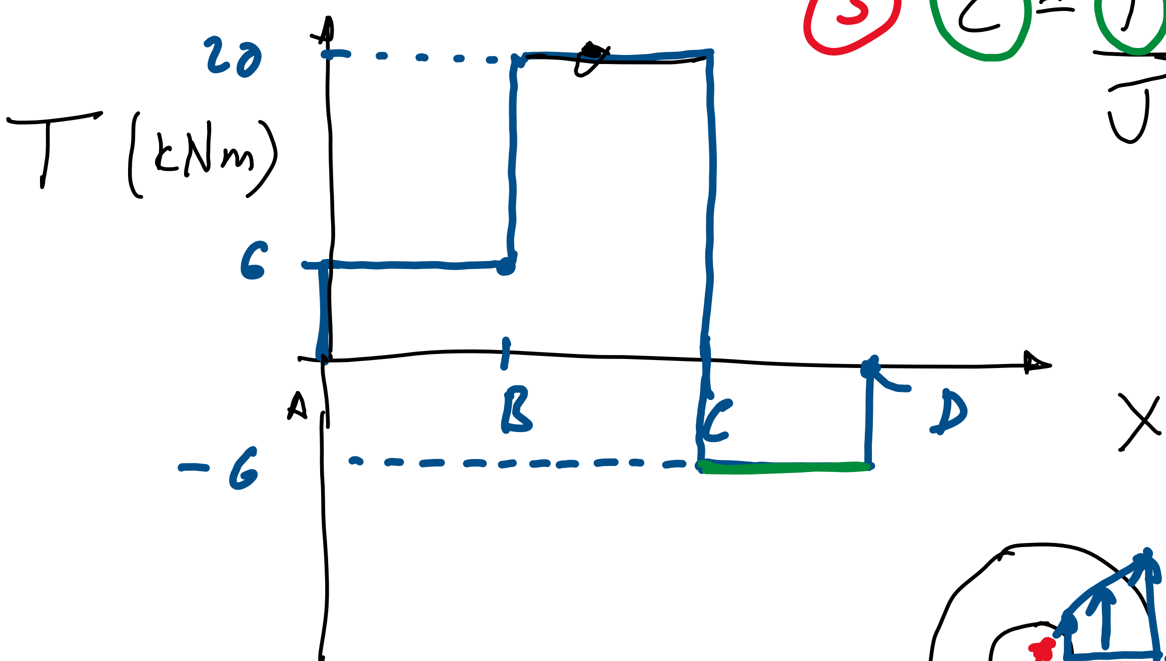


SFD & BMD

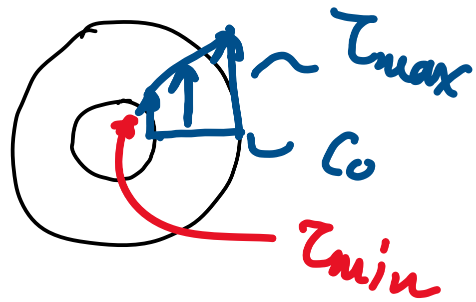
Torque Diagram



①



③ $\tau = \frac{T\rho}{J}$



② $\tau_{max} = \frac{T c_o}{J}$

$\tau_{min} = \frac{T c_i}{J}$

$T = 20$ kN-m

$c_o = 60$ mm

$c_i = 45$ mm

$J = \frac{\pi}{32}(d_o^4 - d_i^4)$

$d_o = 120$ mm

$d_i = 90$ mm

$$C_i = 45\ mm \quad \| \quad a_o = 120\ mm$$
$$a_i = 90\ mm$$

J

$J = 13.9\,(10^{-6})\,m^4$

$$\boxed{\begin{array}{l} \tau_{max} = 86.2 \text{ MPa} \\ \tau_{min} = 64.7 \text{ MPa} \end{array}}$$

③ AB & CD will have the same shear stress because they have same d and the same torque (6 kN.m)

$$\tau_{AB} = \tau_{CD} = \frac{T\,C}{J}$$

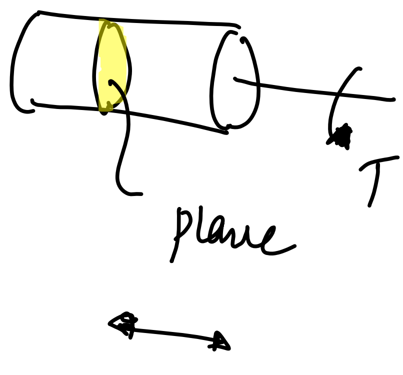
$$65\,(10^6) = \frac{6\,(10^3)\,(\cancel{d}/2)}{\frac{\pi\, d^{\cancel{4}\,3}}{32}} \text{ ——— Table A-18}$$

$$d = \left(\frac{6\,(10^3)}{65\,(10^6)} \left(\frac{32}{\pi}\right) \right)^{1/3}$$

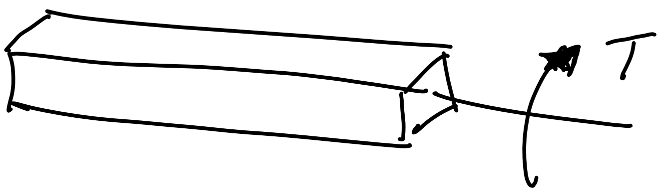
$$\boxed{d = 77.8 \text{ mm}}$$

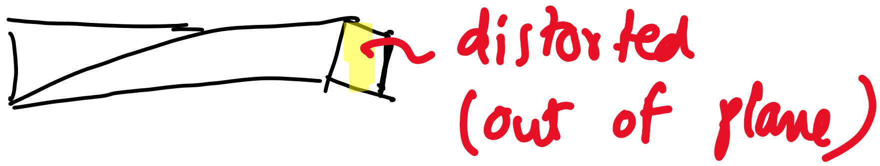
Torsion of rectangular shafts

- circular shafts : plane sections remain plane



- rectangular shaft: plane sections get distorted.

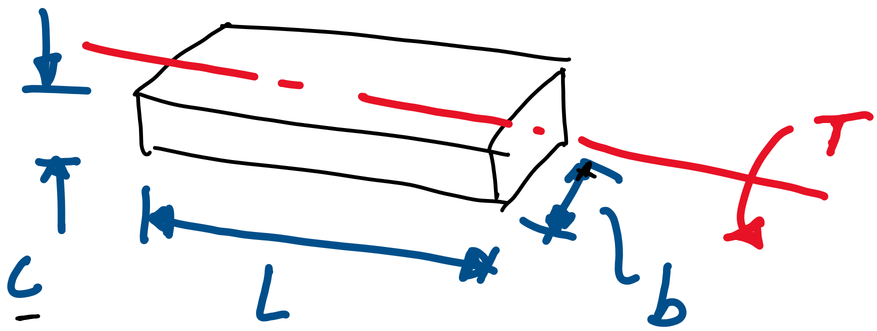




Derivation of τ_{max}, θ is beyond scope of this class.

Table

b/c	1.00	1.50	1.75	2.00	2.50	3.00	4.00	6.00	8.00	10	∞
α	0.208	0.231	0.239	0.246	0.258	0.267	0.282	0.299	0.307	0.313	0.333
β	0.141	0.196	0.214	0.228	0.249	0.263	0.281	0.299	0.307	0.313	0.333



$$\boxed{\tau_{max} = \frac{T}{\alpha b c^2} \qquad \theta = \frac{TL}{\beta b c^3 G}}$$