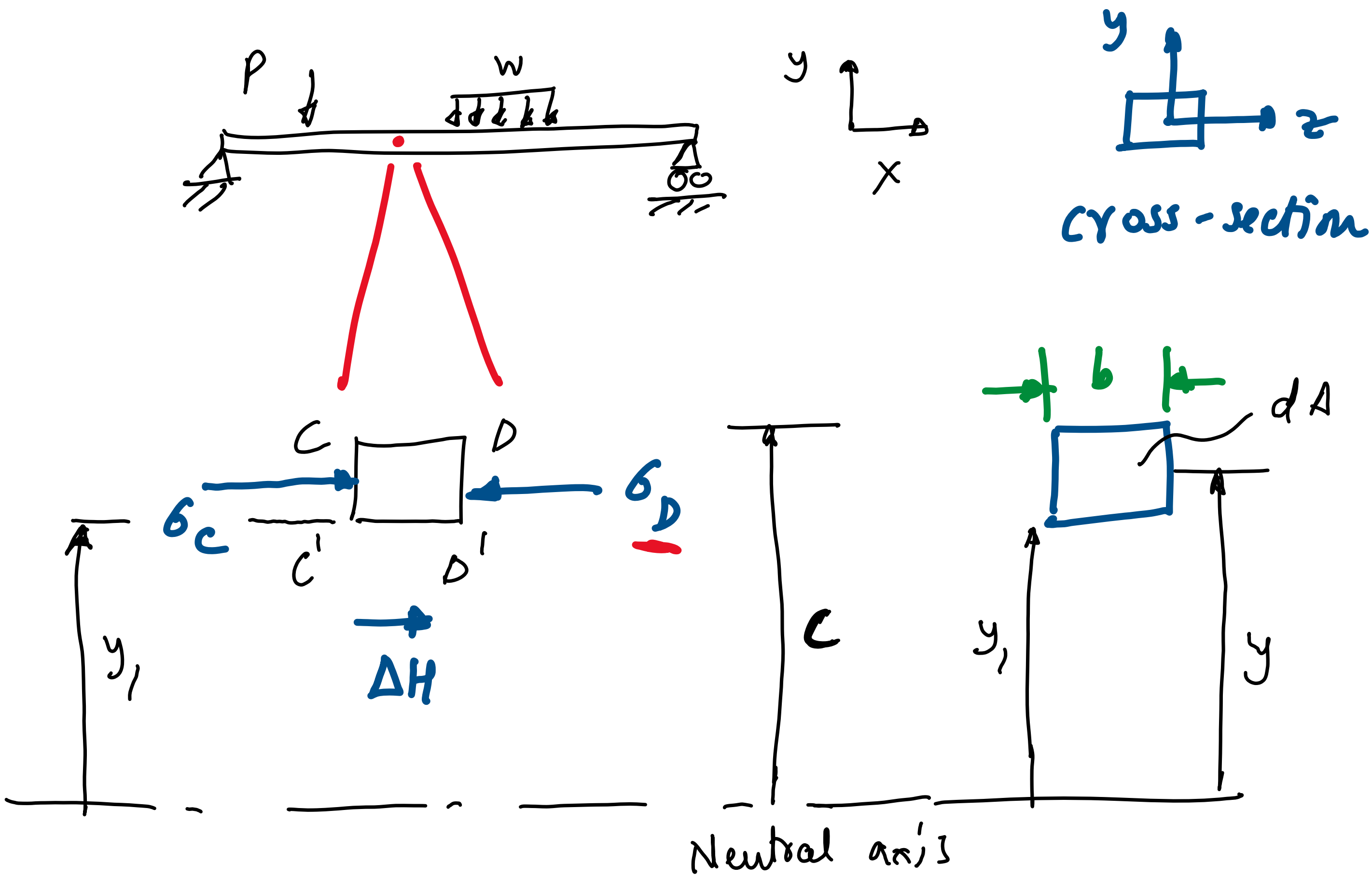
Shear stresses in a beam



ΔH is the force on $C'D'$

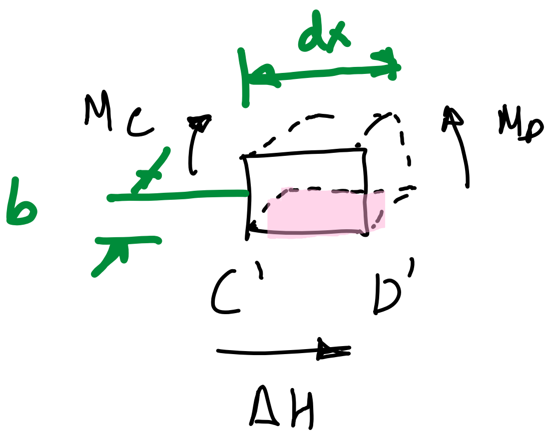
$$\sum F_x = \Delta H + \int \sigma_C \, dA - \int \sigma_D \, dA = 0$$

$$\Delta H = \int (\sigma_D - \sigma_C) \, dA$$

But we know that $\sigma = \dfrac{My}{I}$

$$\Delta H = \int \left(\frac{M_D y}{I} - \frac{M_C y}{I} \right) dA$$

$$\Delta H = \int \left(\overbrace{\frac{M_D\, y}{I}}^{\sigma} - \frac{M_C\, y}{I} \right) dA \qquad \text{— (1)}$$



$$\Delta H = \tau\,(b\,dx) \qquad \text{(2)}$$

$$M_D - M_C = dM$$

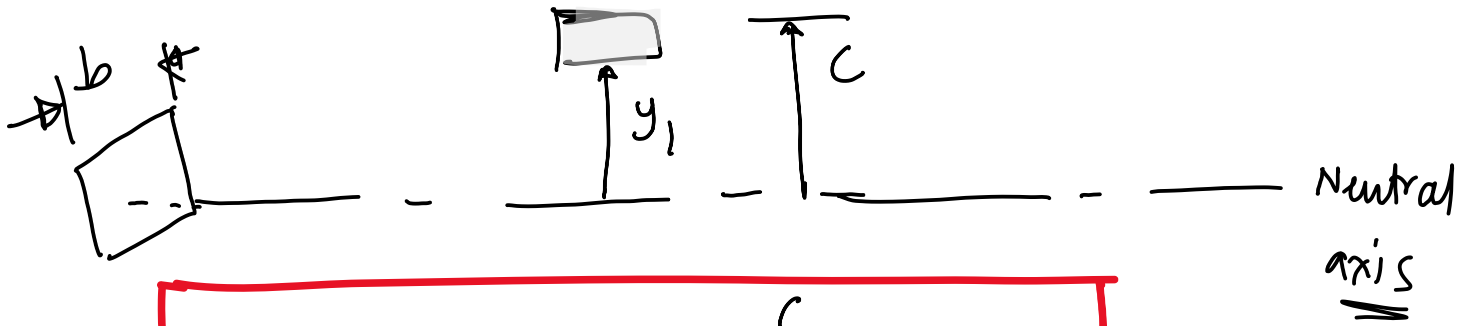
Substitute (2) in (1)

$$\tau\,(b\,dx) = \frac{dM}{I} \int y\, dA$$

$$\tau = \left(\frac{1}{Ib}\right)\left(\frac{dM}{dx}\right) \int y\, dA$$

But $dM/dx = V$ (see BMD/SFD notes)

$$\boxed{\tau = \frac{VQ}{Ib} \quad \text{where} \quad Q = \int y\, dA}$$



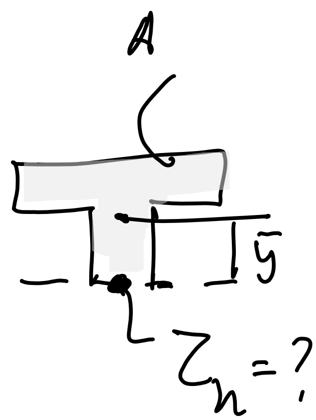
$$\boxed{\tau = \frac{VQ}{Ib} \qquad Q = \int_{y_1}^{c} y\, dA}$$

2 Cases

① on the neutral axis $y_1 = 0$

$$Q = \int_{0}^{c} y\, dA \quad \text{(Max)}$$

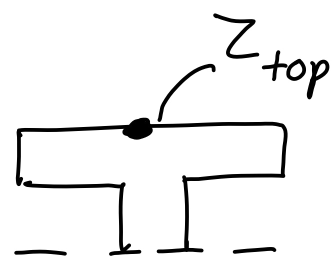
$$= \bar{y} A$$



τ_n is max at neutral axis

② on the top edge $y_1 = c$

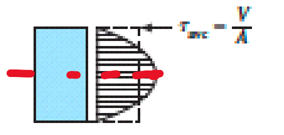
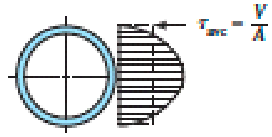
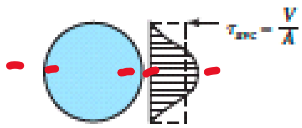
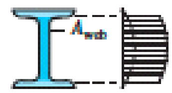
$$Q = \int_{c}^{c} y\, dA = 0$$



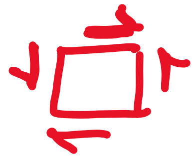
τ_{top} is zero on the top edge

Table 3–2

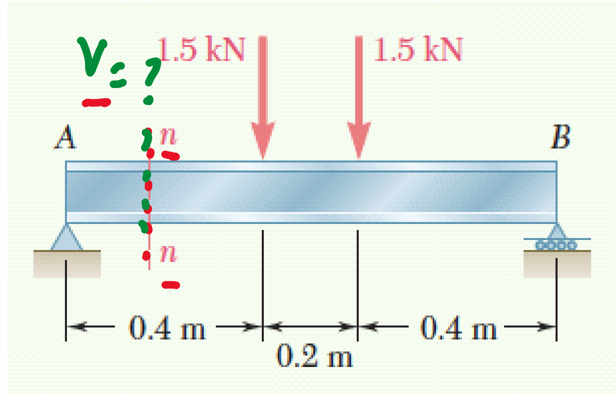
Formulas for Maximum Transverse Shear Stress from VQ/Ib

Beam Shape	Formula	Beam Shape	Formula
 Rectangular	$\tau_{max} = \dfrac{3V}{2A}$	 Hollow, thin-walled round	$\tau_{max} = \dfrac{2V}{A}$
 Circular	$\tau_{max} = \dfrac{4V}{3A}$	 Structural I beam (thin-walled)	$\tau_{max} \approx \dfrac{V}{A_{web}}$

Q1



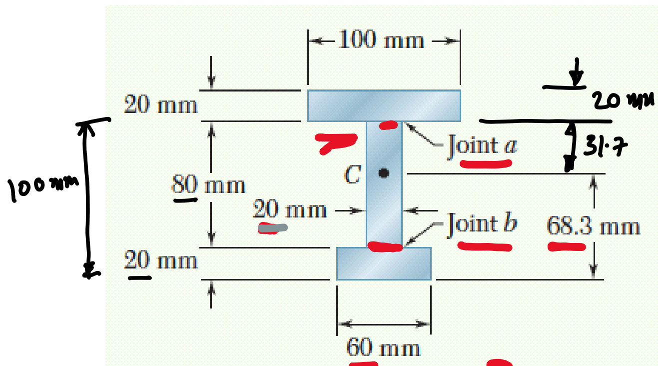
The beam is simply supported and loaded as shown. The beam is made up of three plates with two joints, Joint A and Joint B, as shown. The width of the beam is 20 mm. The location of the centroid of the section is at C as shown. The centroidal moment of inertia is 8.63 x 10^(-6) m^4. Find the average shearing stress at Joint A and Joint B at the section n-n



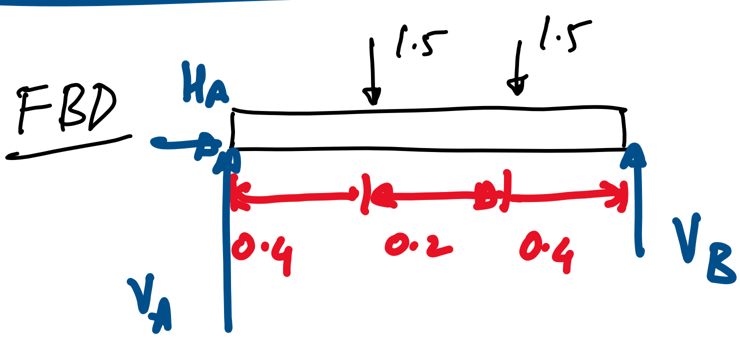
$I = 8.63\,(10^{-6})\,m^4$

(n-n)

$$\tau = \frac{VQ}{Ib} \qquad Q = \int_{y_1}^{c} y\, dA$$

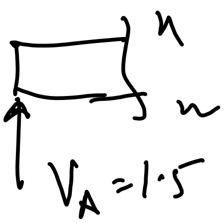


$= 100 - 68.3$
$= 31.7$



$H_A = 0$; $V_A = V_B = 1.5$ kN (Because of symmetry)

Shear force at n-n

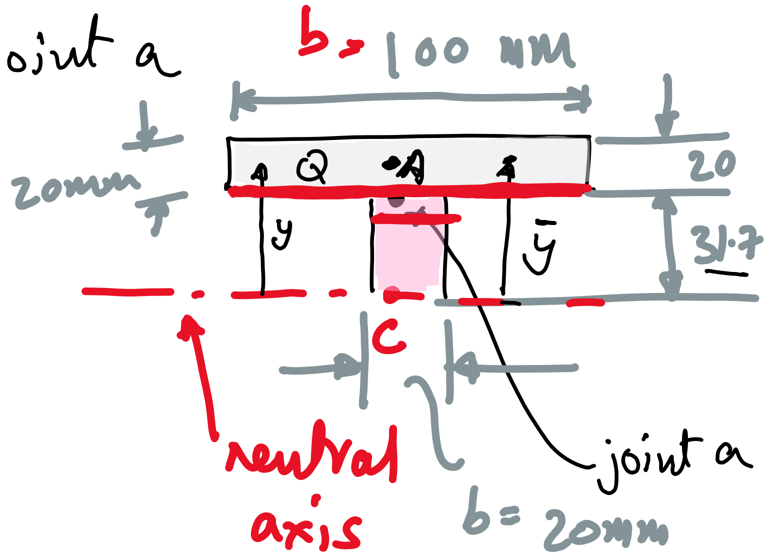


$V_A - V = 0$

$1.5 - V = 0$

$V = 1.5$ kN

(i) Shear stress at joint a



$$\tau_a = \frac{V\,Q}{I\,b}$$

1.5 kN

8.63 (10^{-6})

$$Q = \int_{y_1}^{c} y\,dA = \int_{31.7}^{51.7} y\,dA$$

$$= \bar{y}A = \underbrace{(31.7+10)}_{\bar{y}}(10^{-3})\left[\underbrace{(100)(20)}_{A}\,10^{-6}\right]$$

convert to m

convert to m

$$= 83.4\,(10^{-6})\,m^3$$

$b = 20$ mm

$$\underbrace{\tau \propto \frac{1}{b}}$$

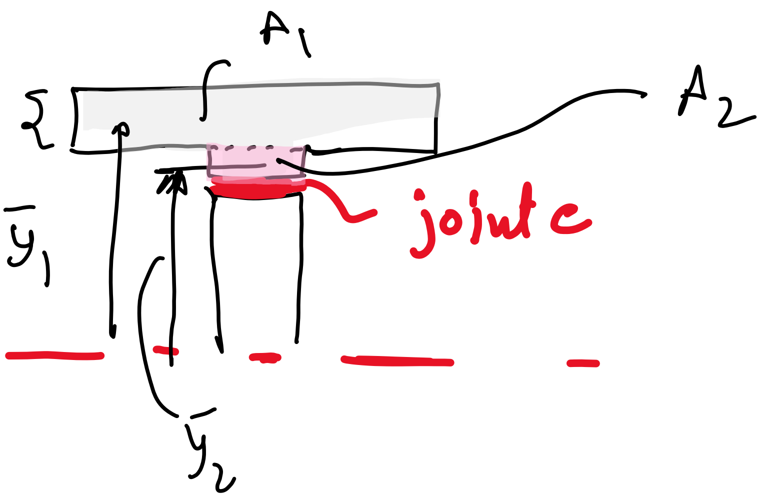
small b
large τ

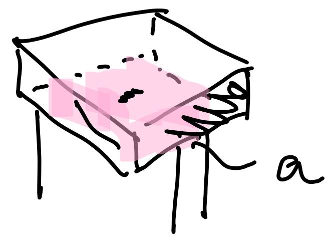
Choose a smaller b.

$b \Rightarrow 20, 100$

smaller value

$$\tau_a = \frac{(1.5)(10^3)(83.4)(10^{-6})}{(8.63)(10^{-6})(20)(10^{-3})} \Rightarrow \boxed{\tau_a = 725 \text{ kPa}}$$





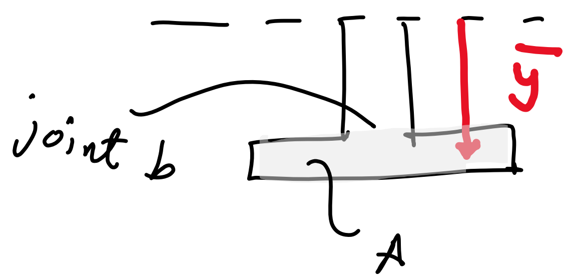
$$\int y\,dA = \bar{y}A = \underline{y_1A_1} + \underline{y_2A_2}$$

Solve for τ_b.

$$\tau_b = \frac{VQ}{Ib}$$

$Q = A\bar{y} = 70\,(10^{-6})\,m^3$

b = 20mm

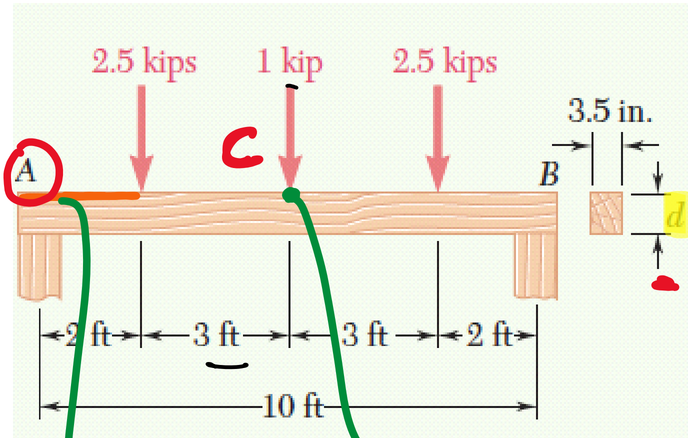


$$\boxed{\tau_b = 608 \text{ kPa}}$$

Q2

max stress

A beam AB supports three loads as shown. The material used for the beam has a normal stress (sigma) = 1800 psi and shear stress (tau) = 120 psi. Determine the minimum depth d such that it meets strength constraints specified by the given stresses



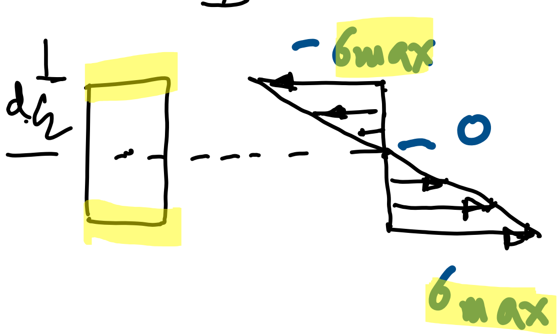
V_{max} τ_{max}

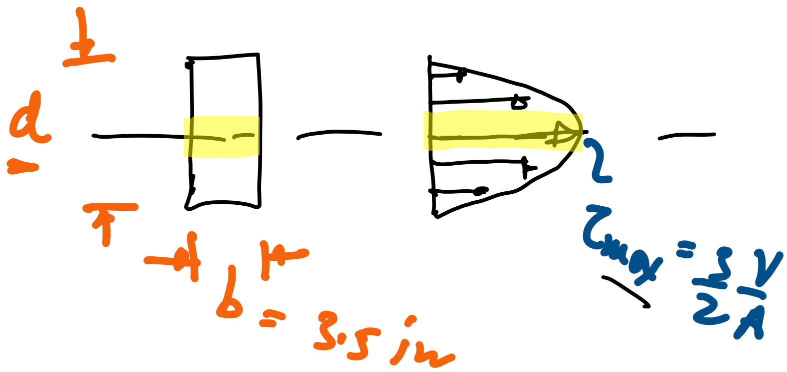
M_{max} σ_{max}

$\sigma_{max} = 1800$ psi ✓

$\tau_{max} = 120$ psi ✓

$$\sigma = \frac{My}{I} \; ; \qquad \tau = \frac{VQ}{Ib}$$





$$\sigma_{max} = \frac{M \, d/2}{bd^3/12} \quad \text{— A-18}$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$

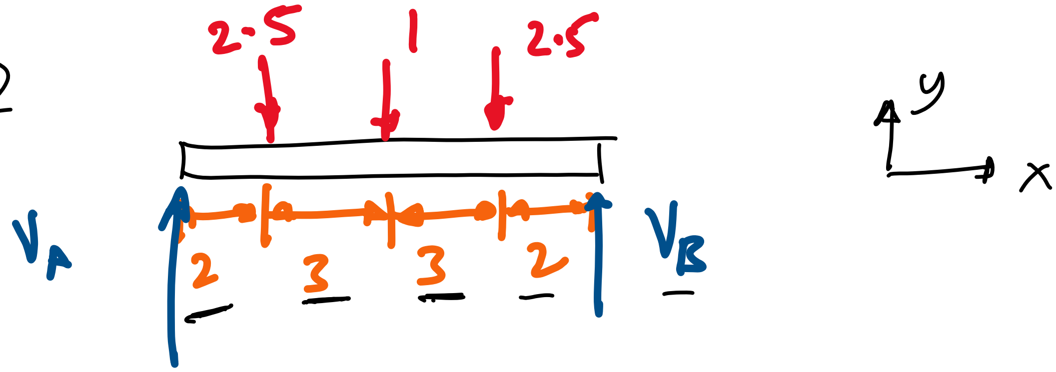
$$\boxed{\sigma_{max} = \frac{6M_{max}}{bd^2}} \qquad \boxed{\tau_{max} = \frac{3}{2}\frac{V_{max}}{bd}}$$

We need V_{max}, M_{max}

We need V_{max}, M_{max}

① Draw V–x and M–x curve to compute V_{max} and M_{max}

FBD



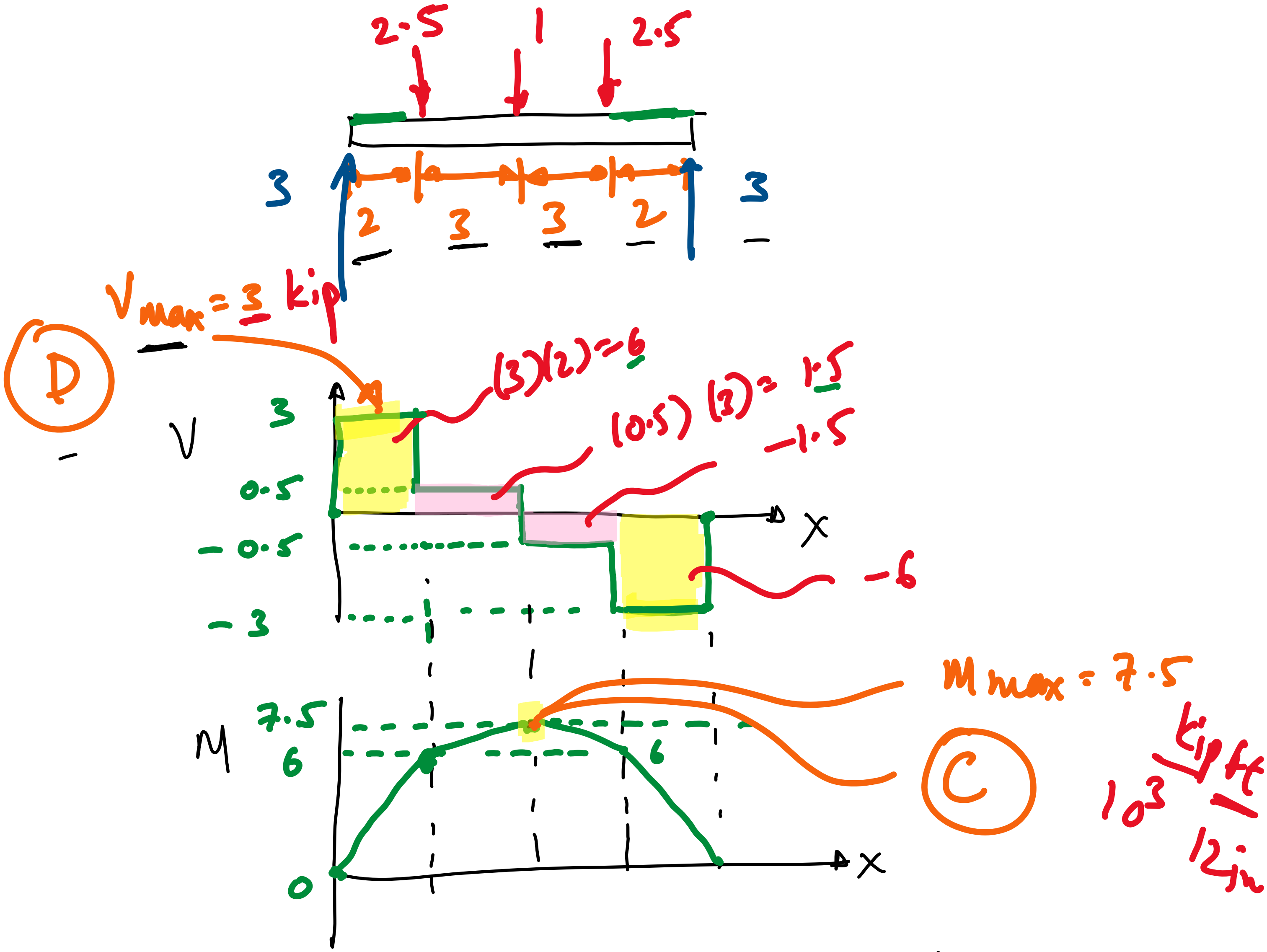
$\Sigma F_y = 0 \Rightarrow V_A + V_B - 2.5 - 1 - 2.5 = 0$

$V_A + V_B = \underline{6}$

$\Sigma M_A = 0 \quad -(2.5)(2) - (1)(5) - (2.5)8 + V_B(10) = 0$

$V_B = \dfrac{5 + 5 + 20}{10} = 3$ kips

$V_A = 3$ kips.



$$\sigma_{max} = \frac{6\,M_{max}}{b d^2} \qquad \tau_{max} = \frac{3\,V_{max}}{2\,b d}$$

C is critical

D is critical

(ii) Design for d

(a) Use $\sigma = 1800$ psi at (C) to compute d

check if τ at (D) < 120 psi

If this checks out d is good

If not go to (b)

(b) Use $\tau = 120$ psi at (D) to compute d. Check if σ at (C) < 1800 psi

If this checks out d is good.

(a) Use $\sigma = 1800\,psi$ at (C) to design d

$$\sigma_{max} = \frac{6M_{max}}{bd^2}$$

$$1800 = \frac{6(7.5)(10^3)(12)}{(3.5)\,d^2}$$

ft to in

kip

$$d = 9.258\ in$$

Check if τ at (D) is < 120 psi

$$\tau = \frac{3}{2}\frac{V_{max}}{bd} = \frac{3}{2}\frac{(3000)}{(3.5)(9.258)}$$

$$\tau = 138.8\ psi > \underline{120\ psi}$$

max allowed τ

If d = 9.258 the beam will <u>fail</u> in shear at (D)

(X)

(b) Use $\tau = 120$ psi at (D) to design d

$$\tau_{max} = \frac{3}{2}\frac{V_{max}}{bd}$$

$$120 = \frac{3}{2}\frac{3000}{(3.5)d}$$

$$d = \underline{10.714} \text{ in}$$

Check if σ at (C) is < 1800 psi

$$\sigma = \frac{6M_{max}}{bd^2} = 6\frac{(7.5)(10^3)(12)}{(3.5)(10.714)^2}$$

$$\sigma = 1344.07 \text{ psi} < 1800 \text{ psi}$$

Since $\sigma < 1800$ psi DESIGN is ACCEPTABLE

$$\boxed{d = 10.714 \text{ in}}$$ Final Design