Where we are / and / where we are going

① FBD + SFD + BMD

↓ V, M

② Normal / Shear stress

NEXT

↓ $\sigma_x, \sigma_y, \tau_{xy}$

③ Mohr's circle

↓ $\sigma_a, \sigma_B, \sigma_c \rightarrow \sigma_1 > \sigma_2 > \sigma_3$

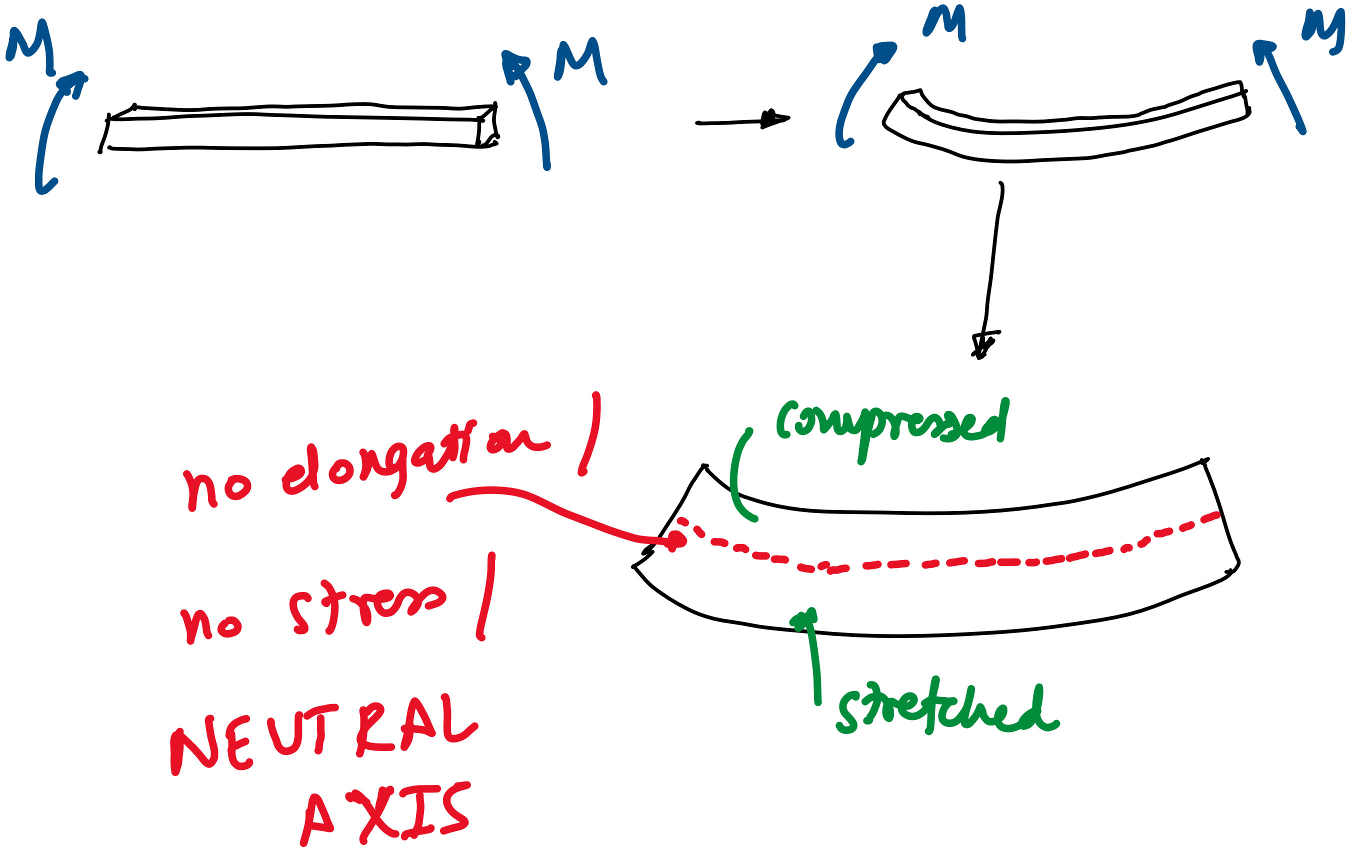
④ Design based on different theories

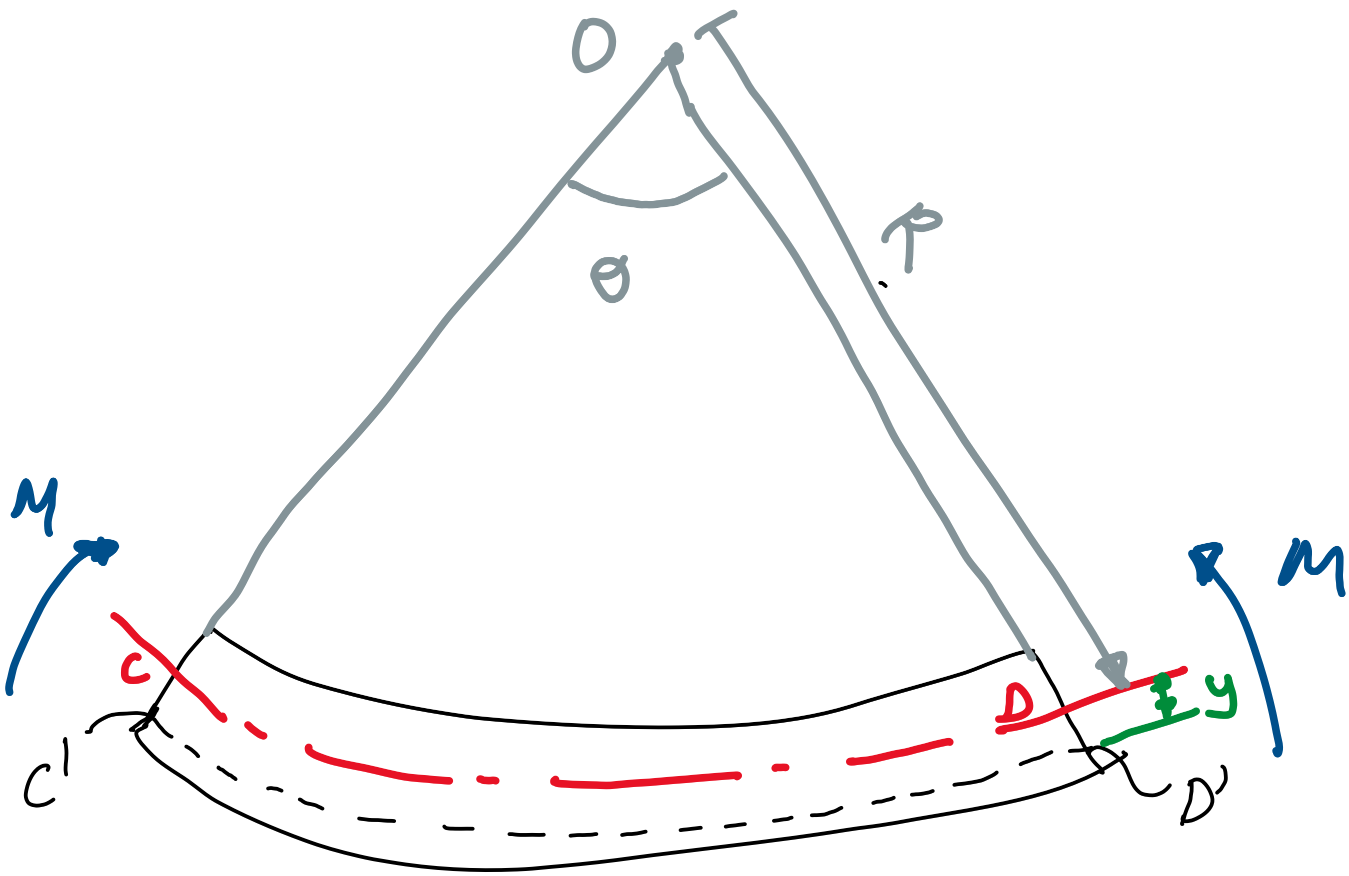
e.g. max. shear stress theories

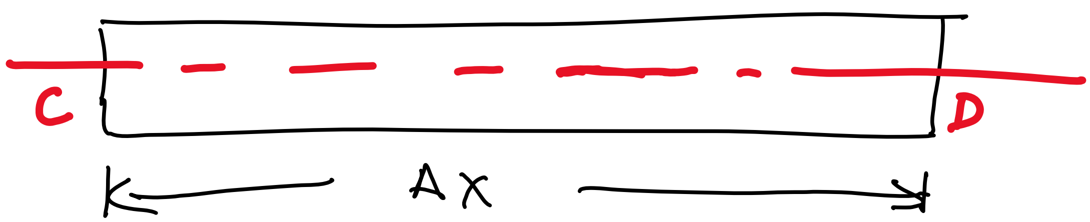
Bit later

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \quad \Big\} \text{ determines failure}$$

Normal stresses in a beam







$$\varepsilon = \frac{C'D' - CD}{CD}$$

$$CD = R\theta \qquad C'D' = (R+y)\theta$$

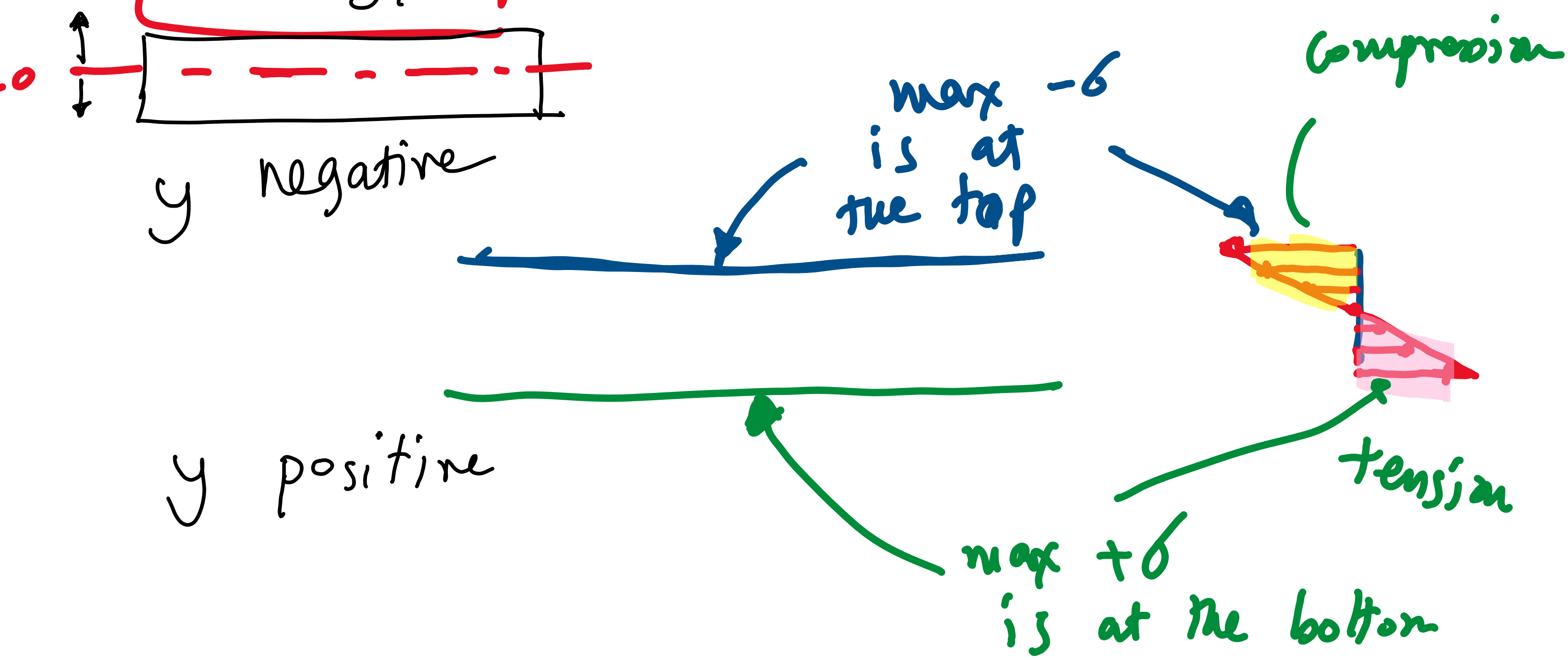
$$\varepsilon = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

$$\sigma = E\varepsilon = Ey$$

$$\sigma = E\,\varepsilon = \frac{E\,y}{R}$$

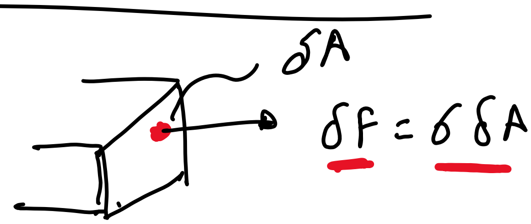
$$\boxed{\begin{aligned}\varepsilon &= y/R \\ \sigma &= Ey/R\end{aligned}}$$

$$\sigma \propto y$$



---

① Net force



$$\delta F = \sigma \, \delta A$$

$$F = \int \delta F = \int_A \sigma \, \delta A = \int_A \frac{Ey}{R} \, \delta A$$

$$F = \frac{E}{R} \int y \, \delta A \Rightarrow \int y \, \delta A = 0$$

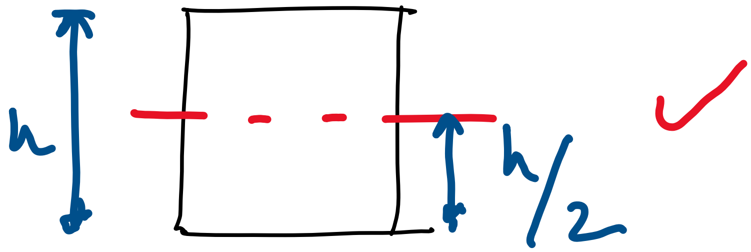
$$\text{Centroid} = \frac{\int y \, \delta A}{\int \delta A} = 0$$

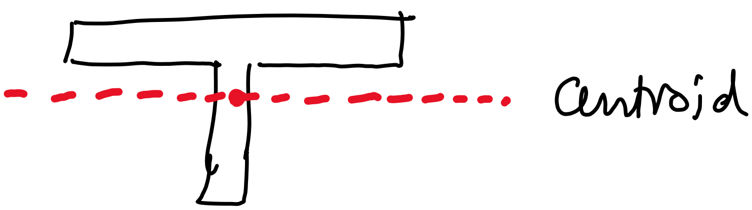
F=0 because there is no external force on the beam

- on the beam

$$\text{Centroid} = \frac{\int y\,\delta A}{\int \delta A} = 0$$

[IMPLIES THAT the neutral axis is the centroid of the beam]



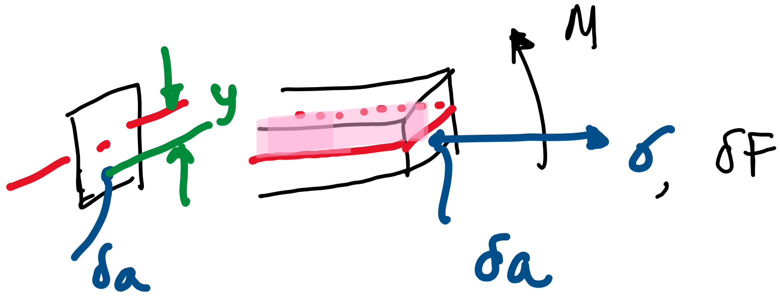


$$\bar{y} = \frac{\int y\,\delta A}{\int \delta a}$$

compute the centroid in order to locate neutral axis

② Net moment

Moment about the neutral axis



$$M = \int \delta F \, y$$

$$M = \int (\sigma \, \delta a) \, y$$

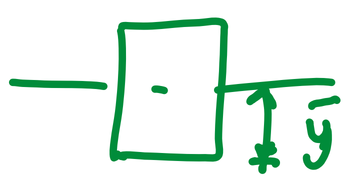
$$M = \int \left(\frac{E}{R} y \, \delta a\right) y \qquad \sigma = \frac{E}{R} y \; *$$

$$M = \frac{E}{R} \underbrace{\int y^2 \delta a}_{\text{moment of inertia } (I)}$$

$$\underbrace{M = \left(\frac{E}{R}\right) I \qquad \text{Also, } \sigma = \left(\frac{E}{R}\right) y}_{\text{Combine}}$$

$$(*) \quad \boxed{\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}}$$

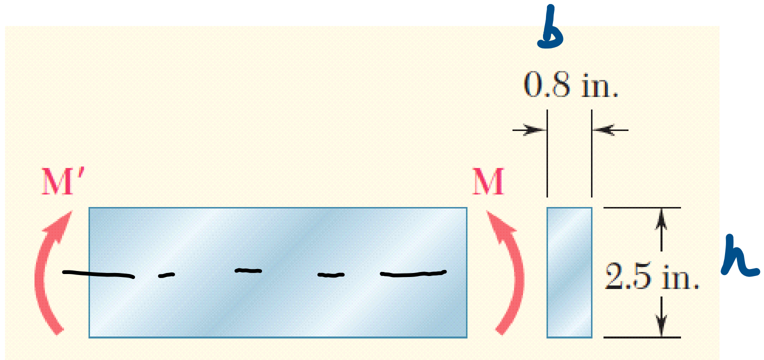
$\frac{I}{\bar{y}} \rightarrow$ section modulus



Q1

A steel bar of 0.8 x 2.5 in is subject to a moment M as shown. Find M that causes the bar to yield. Given sigma_yield = 36 ksi

σ_y

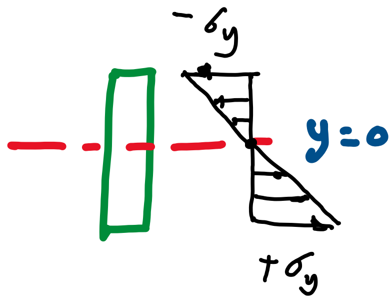


$M = ?$

$\sigma_y = 36\ \underbrace{k}si$

kilopound per sq. inch

$$\frac{M}{I} = \frac{\sigma_y}{y}, \quad y = h/2 \qquad \Rightarrow \quad M = \sigma_y \frac{I}{y}$$



See table A-18 for $I = \frac{bh^3}{12}$

Max stress at $y = h/2$

$$M = \sigma_y \left(\frac{1}{12}\right) \frac{bh^3}{h/2} = \frac{\sigma_y b h^2}{6}$$

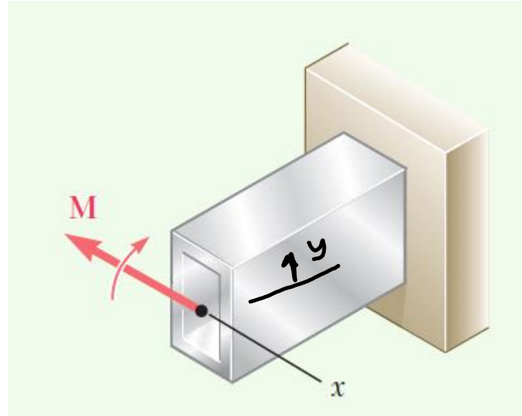
$$M = \frac{(36)(0.8)(2.5)^2}{6}$$

$$\boxed{M = 30\ \text{kip} \cdot \text{in}}$$

$\text{kip} = 1000\ \text{lbf}$

Q2

A rectangular tube has an ultimate strength of 60 ksi and dimensions as shown. Find (a) bending moment M such that factor of safety is 3, and (b) radius of curvature of the tube assuming E = 10.6 x 10^6 psi



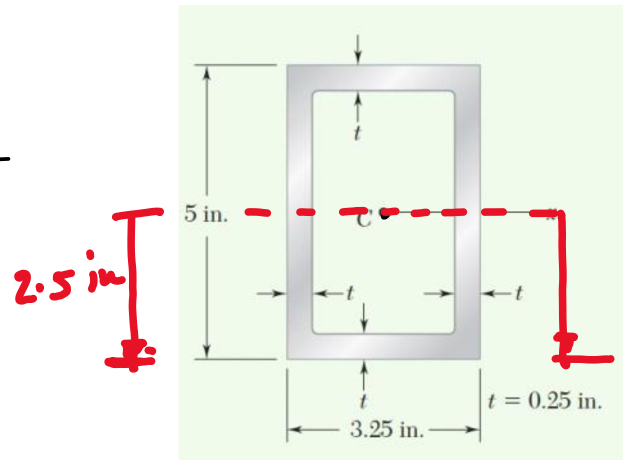
$S_{ut} = 60$ ksi

$n = 3$

(a) $M = ?$

(b) $\frac{1}{R} = ?$ $E = 10.6\,(10)^6$ psi

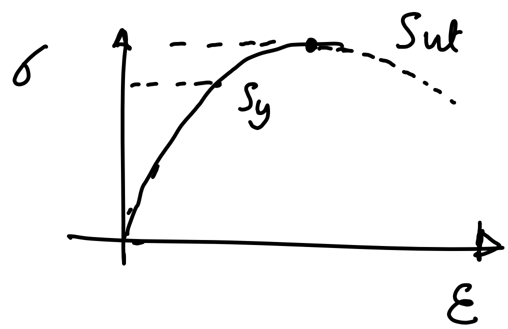
(a) $\frac{M}{I}^{?} = \frac{\sigma}{y}^{?}$ (allowable stress)



$n = \frac{\text{Max. stress}}{\text{Allowable stress}} = \frac{S_{ut}}{\sigma}$

$3 = \frac{60}{\sigma}$

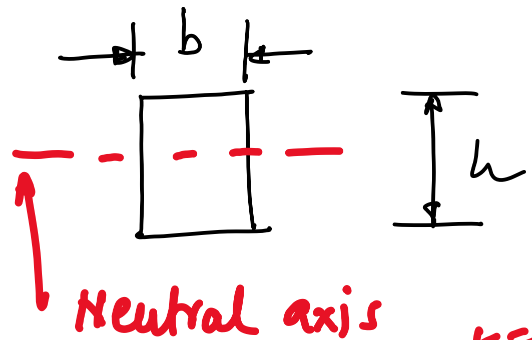
$\Rightarrow \sigma = 20$ ksi

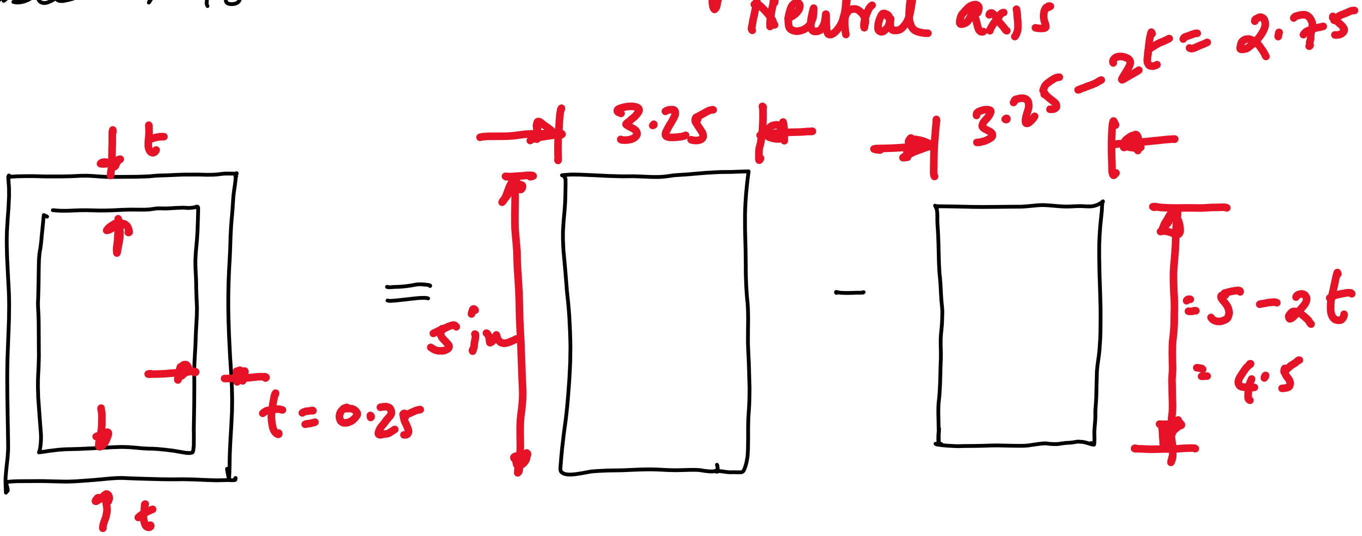


$y = 2.5$ in

$I = \frac{bh^3}{12}$ ✓ —(1)

Table A-18





$$I = I_1 - I_2$$

$$I = \frac{(3.25)(5)^3}{12} - \frac{(2.75)(4.5)^3}{12} \quad \left[\text{From } (1)\right]$$

$$I = 12.97 \ in^4$$

$$M = \sigma\left(\frac{I}{y}\right) = (20)\frac{(12.97)}{(2.5)} = 103.8$$

$$\boxed{M = 103.8 \ kip.in}$$

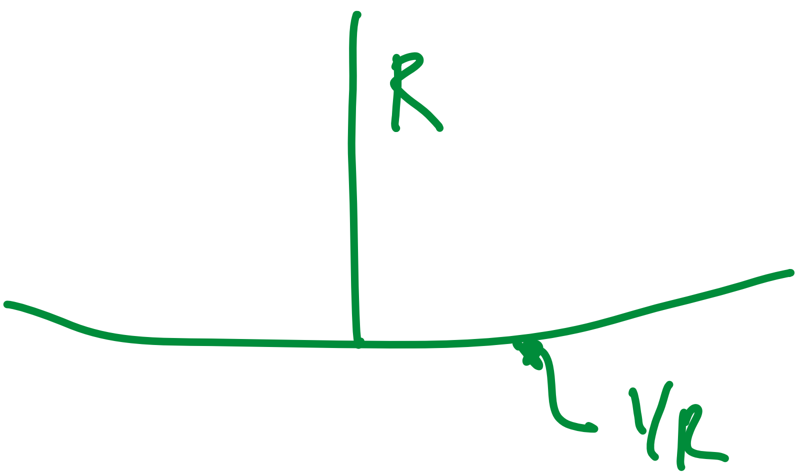
(b) $$\left.\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}\right\}$$

$$\frac{1}{R} = \text{radius of curvature} = \frac{\sigma}{yE} = \frac{20}{(2.5)(10.6(10^6))}$$

$$\boxed{\frac{1}{R} = 7.55(10^{-4})\ in^{-1}}$$

1/R is small

R is big (~ flat)

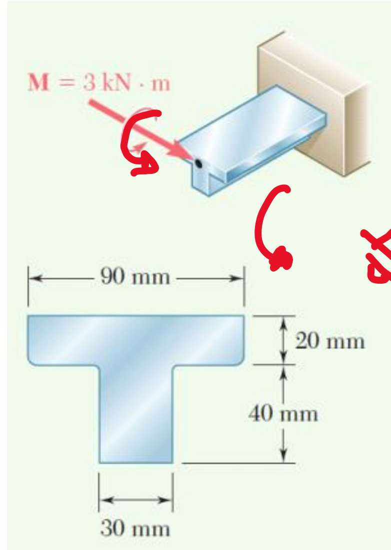


$R = \infty$

$\frac{1}{R} \approx 0$

Q3

The beam shown is acted upon by a moment M = 3 kNm. Determine the maximum tensile and compressive stress on the beam



$M = 3\ kN\text{-}m$

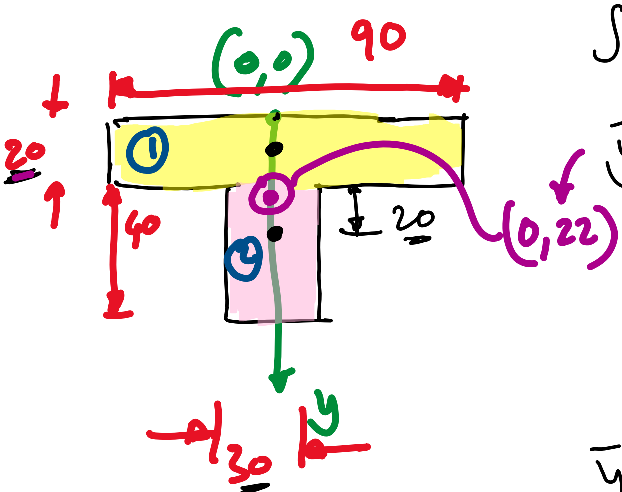
$\sigma_T = ?\quad \sigma_c = ?$

$$\boxed{\frac{M}{I} = \frac{\sigma}{y}}\qquad \frac{\sigma_c, \sigma_T}{y_c, y_T}$$

(neutral axis?)
NA

---

① Neutral axis ? $= \dfrac{\int y\,dA}{\int dA}$

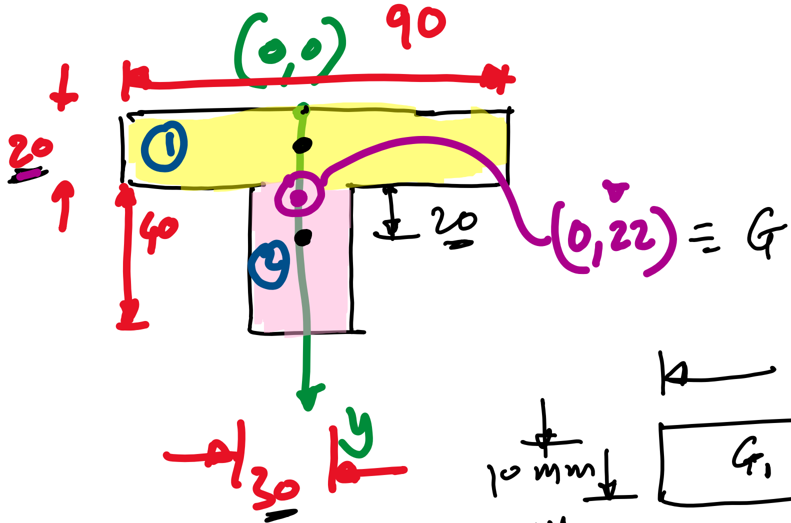


$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2}$$

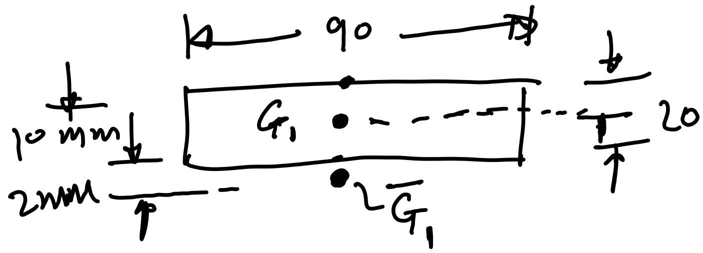
$$= \frac{(10)\left[(90)(20)\right] + (40)\left[(30)(40)\right]}{(90)(20) + (30)(40)}$$

$\bar{y} = 22\ mm$

② Compute I



$I_{\bar{G}} = ?$



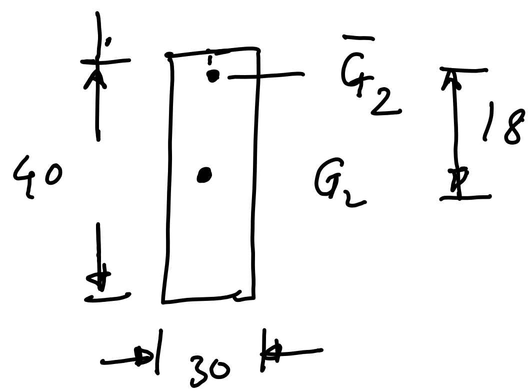
$$I_{\bar{G}_1} = I_{G_1} + \left(r^2_{G_1\bar{G}_1}\right) A_1$$

~ parallel axis theorem

$$= \frac{1}{12}(90)(20)^3 + (12)^2(90)(20)$$

$$I_{\bar{G}_2} = I_{G_2} + r^2_{G_2\bar{G}_2} A_2$$

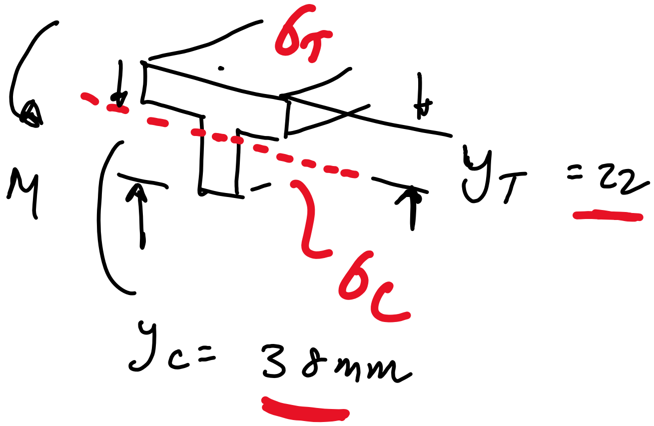
$$= \frac{1}{12}(30)(40)^3 + (18)^2(40)(30)$$



$$I_G = I_{\bar{G}_1} + I_{\bar{G}_2}$$

$$= 8.68\,(10^{-7})\ m^4$$

$I = I_G = 8.68\,(10^{-7})\,m^4$



$$\frac{M}{y} = \frac{\sigma}{I}$$

① $$\sigma_C = \frac{M y_C}{I}$$

$$= \frac{3\,(10^3)\;\;38\,(10^{-3})}{8.68\,(10^{-7})}$$

$$\boxed{\sigma_C = 131.3\ MPa}$$

② $$\sigma_T = \frac{M y_T}{I}$$

$$= \frac{3(10^3)\,(22)(10^{-3})}{(8.68)(10^{-7})}$$

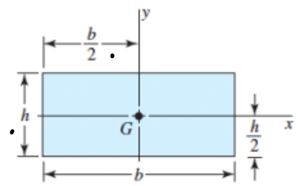
$$\boxed{\sigma_T = 76\ MPa}$$

A-18 (1 of 2)

Part 1 Properties of Sections

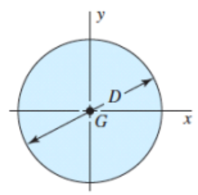
A = area
G = location of centroid
$I_x = \int y^2\, dA$ = second moment of area about x axis
$I_y = \int x^2\, dA$ = second moment of area about y axis
$I_{xy} = \int xy\, dA$ = mixed moment of area about x and y axes
$J_G = \int r^2\, dA = \int (x^2 + y^2)\, dA = I_x + I_y$
= second polar moment of area about axis through G
$k_x^2 = I_x/A$ = squared radius of gyration about x axis

Rectangle



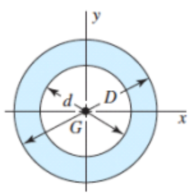
$A = bh$ $\quad I_x = \dfrac{bh^3}{12}$ $\quad I_y = \dfrac{b^3h}{12}$ $\quad I_{xy} = 0$

Circle



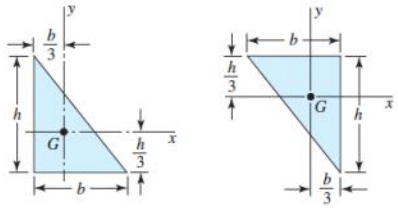
$A = \dfrac{\pi D^2}{4}$ $\quad I_x = I_y = \dfrac{\pi D^4}{64}$ $\quad I_{xy} = 0$ $\quad J_G = \dfrac{\pi D^4}{32}$

Hollow circle



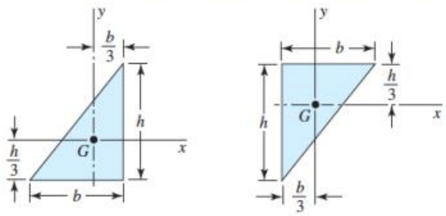
$A = \dfrac{\pi}{4}(D^2 - d^2)$ $\quad I_x = I_y = \dfrac{\pi}{64}(D^4 - d^4)$ $\quad I_{xy} = 0$ $\quad J_G = \dfrac{\pi}{32}(D^4 - d^4)$

Right triangles



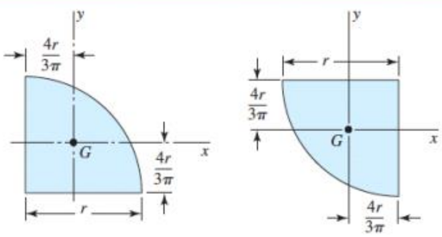
$A = \frac{bh}{2}$ $I_x = \frac{bh^3}{36}$ $I_y = \frac{b^3h}{36}$ $I_{xy} = \frac{-b^2h^2}{72}$

Right triangles



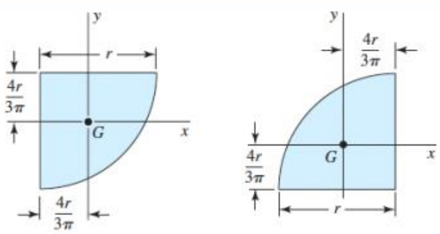
$A = \frac{bh}{2}$ $I_x = \frac{bh^3}{36}$ $I_y = \frac{b^3h}{36}$ $I_{xy} = \frac{b^2h^2}{72}$

Quarter-circles



$A = \frac{\pi r^2}{4}$ $I_x = I_y = r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_{xy} = r^4\left(\frac{1}{8} - \frac{4}{9\pi}\right)$

Quarter-circles



$A = \frac{\pi r^2}{4}$ $I_x = I_y = r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_{xy} = r^4\left(\frac{4}{9\pi} - \frac{1}{8}\right)$