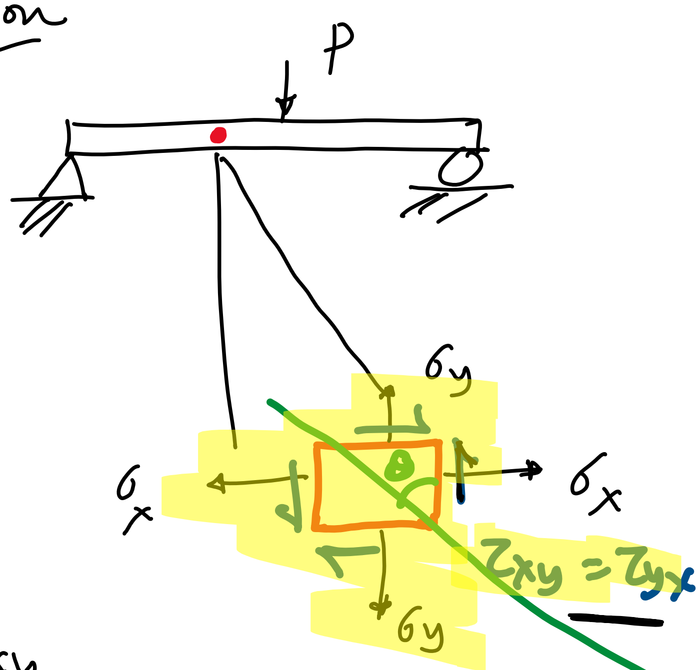
Mohr's circle for plane stress

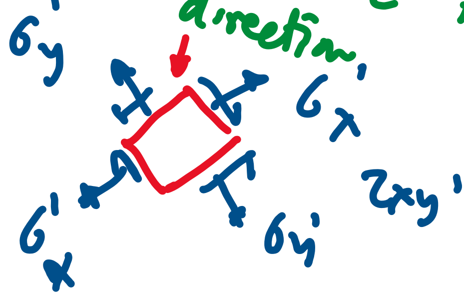
Motivation



We have computed

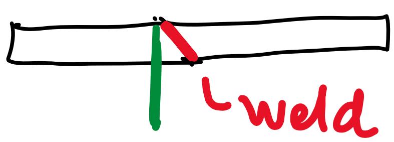
$\sigma_x, \sigma_y, \tau_{xy}$

We want σ, τ in this direction,



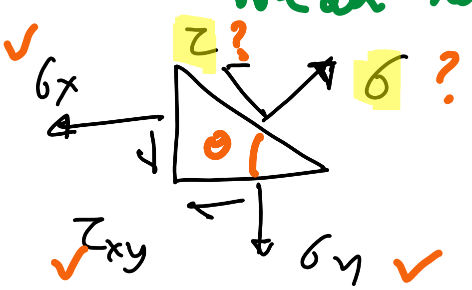
We want to compute

$\sigma_x', \sigma_y', \tau_{xy}'$



weld

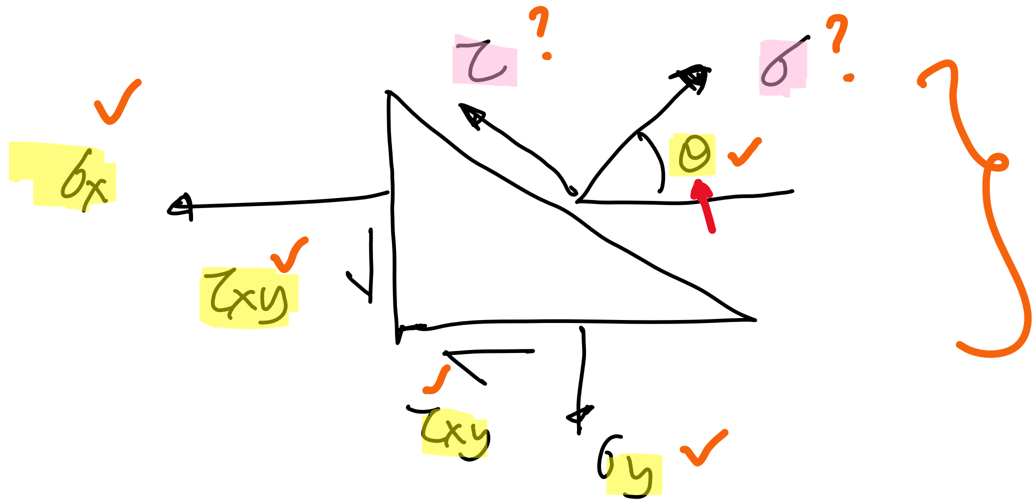
weak here



The worst case stress might be in a different direction.

$\sigma_x, \sigma_y, \tau_{xy} \xrightarrow[\text{circle}]{\text{Mohr's}} \tau, \sigma$ at a given angle θ

$$\boxed{\begin{aligned} \sigma &= \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \underline{\tau_{xy}}\sin 2\theta && \text{(I)} \\ \tau &= -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta && \text{(II)} \end{aligned}}$$



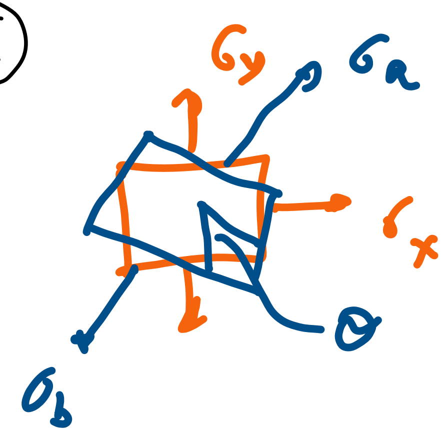
(i) Compute θ for which σ achieves max/min value

Use (I)

a) $\dfrac{d\sigma}{d\theta} = 0$ b) Solve for θ.

c) $\boxed{\tan 2\theta_p = \dfrac{2\tau_{xy}}{(\sigma_x - \sigma_y)}}$ III

direction of the principal stress



Substitute III in I and II

$$\boxed{\begin{gathered}\sigma_{a,b} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ \tau = 0\end{gathered}}$$ III*

$\sigma_{a,b}$ are principal normal stresses

2) compute θ such that τ achieves max/min

a) $\frac{d\tau}{d\theta} = 0$ b) solve for θ

c) $$\boxed{\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}} \quad \text{IV}$$

direction of maximum/minimum shear stress

d) Substitute IV in I and II

$$\boxed{\begin{aligned} \tau_{a,b} &= \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ \sigma &= \frac{\sigma_x + \sigma_y}{2} \end{aligned}} \quad \text{IV}^*$$

Motivation for the Mohr's circle

From (I) and (II)

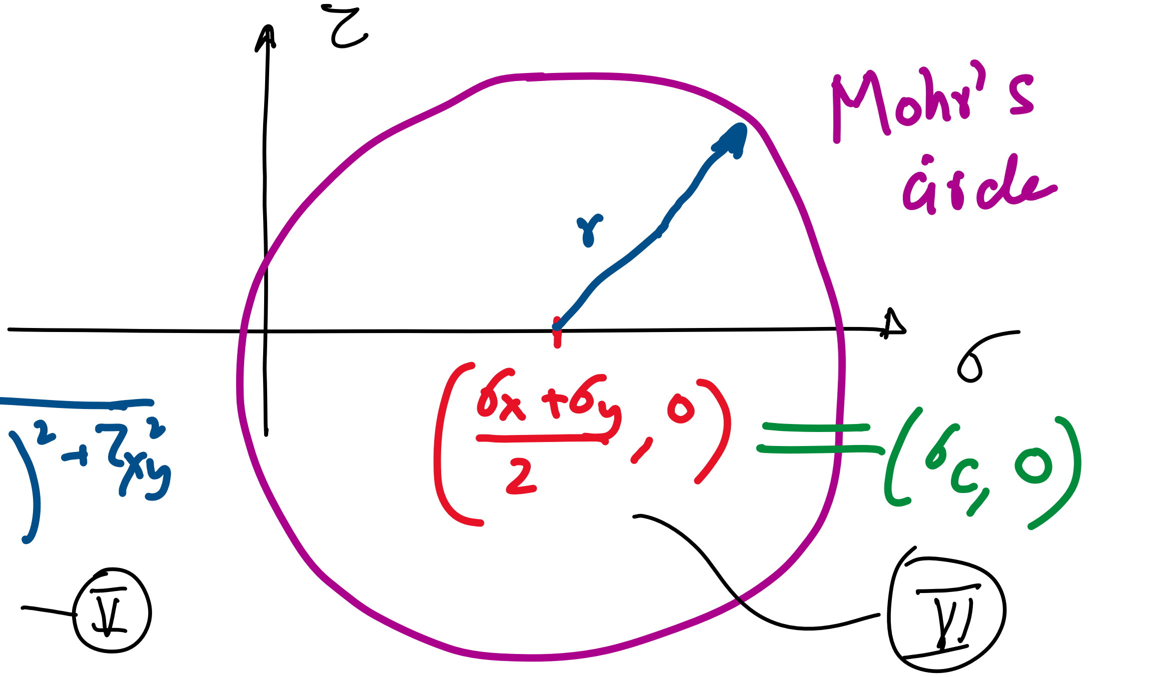
$$\sigma - \left(\frac{\sigma_x+\sigma_y}{2}\right) = \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

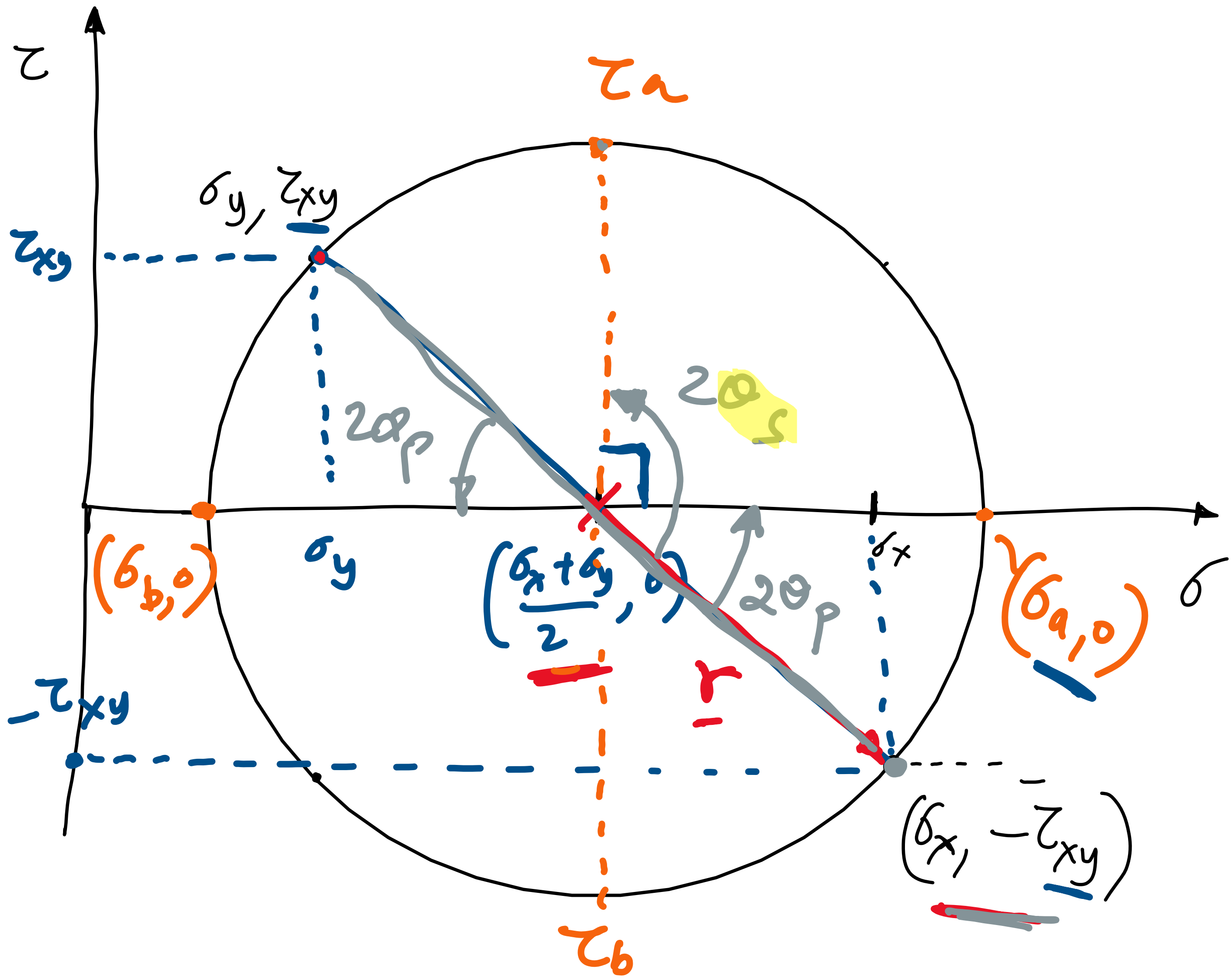
Square the equations and add them up

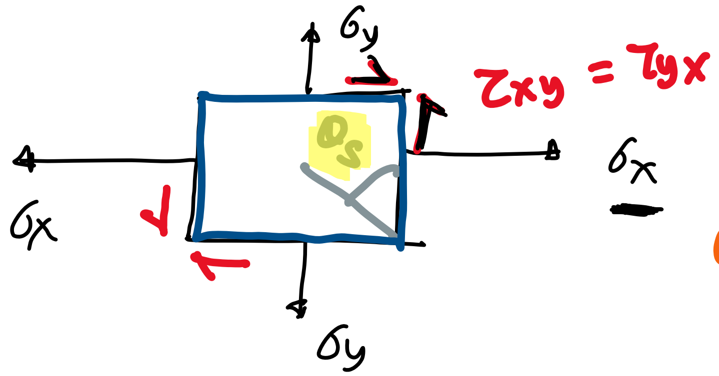
$$\left[\sigma - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right]^2 + \tau^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2 \quad \Bigg\}\ \text{Eqn of a circle}$$

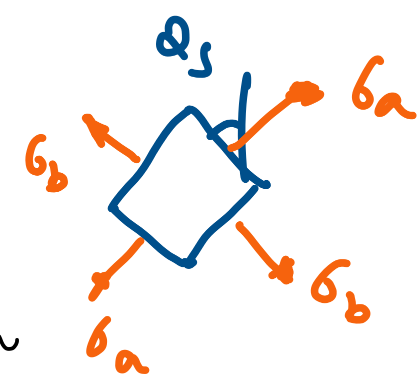
$$(\sigma - \sigma_c)^2 + \tau^2 = r^2$$



$$r = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{— (V)}$$



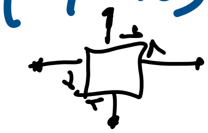




σ: + is tension / − is compression

τ: + CW / − CCW

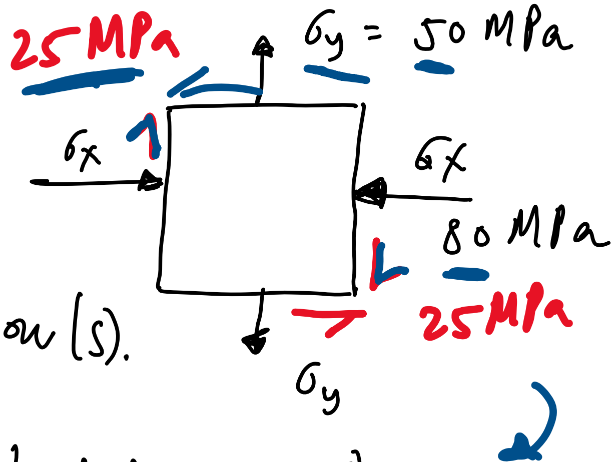
Each point on the Mohr's circle corresponds to an orientation of the stress element (square) to an orientation of the stress element (square)



✓ ① Draw the Mohr's circle

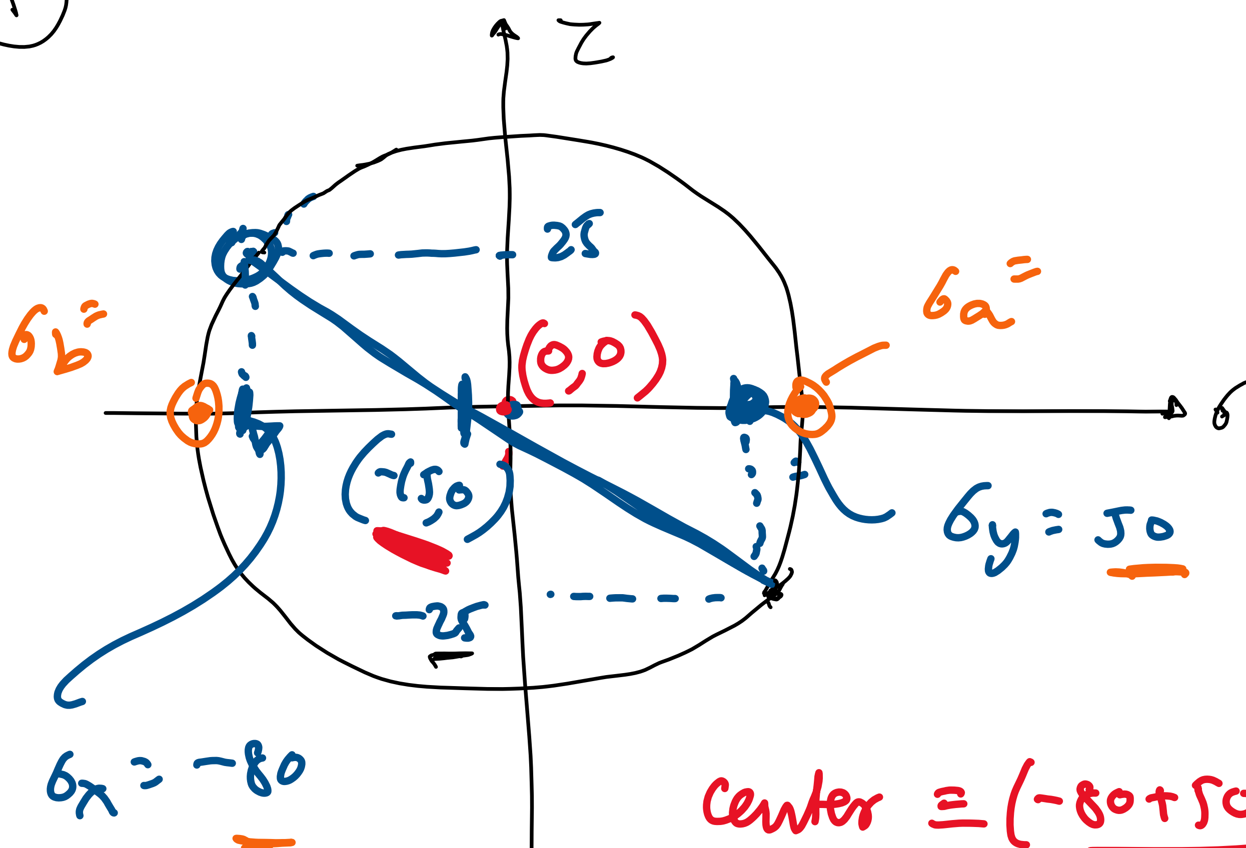
② compute the principle stresses and their direction(s).

③ Compute the maximum/minimum shear stresses and their direction(s).



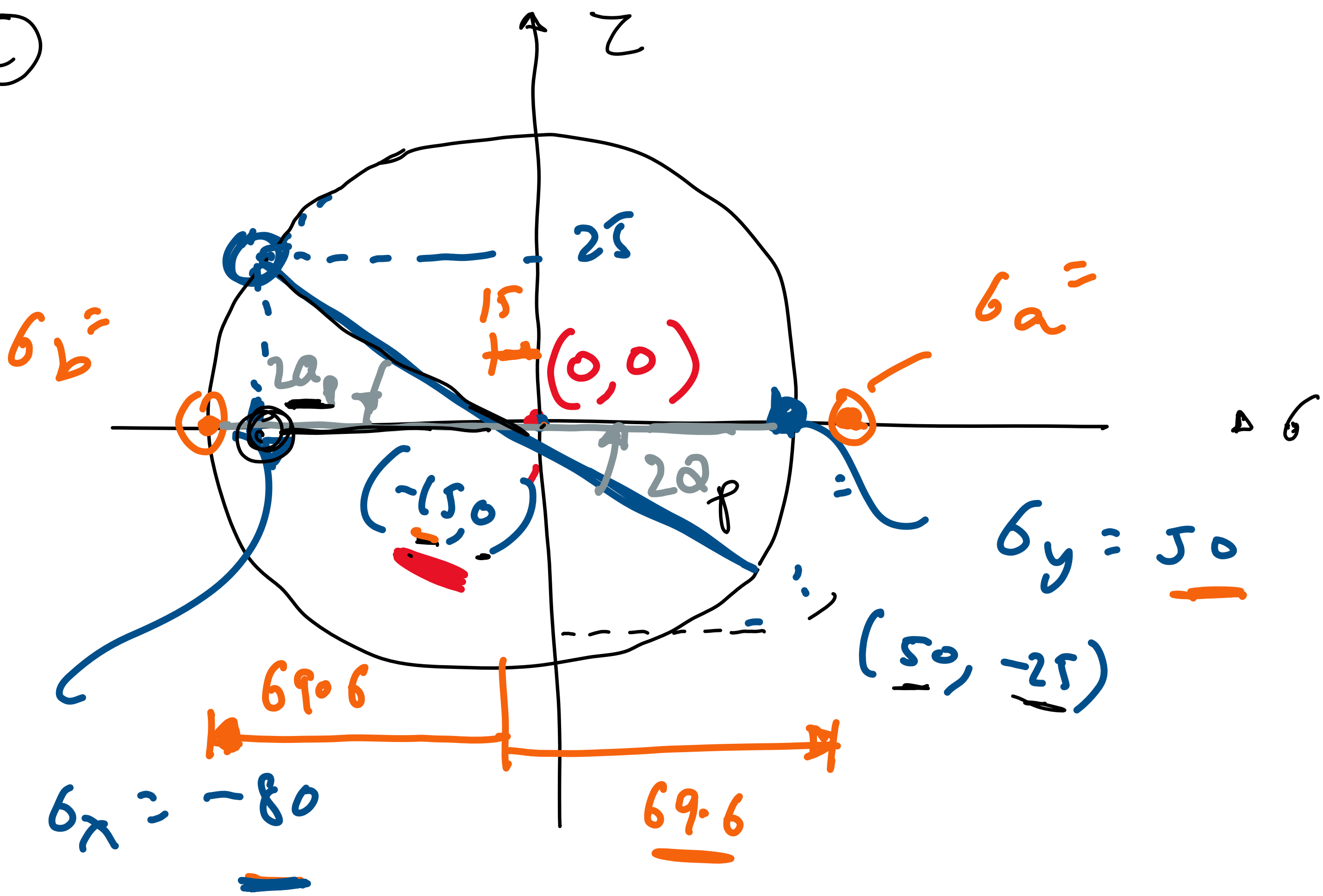
Sign convention: σ +/− tension/compression
τ +/− CW / CCW

①



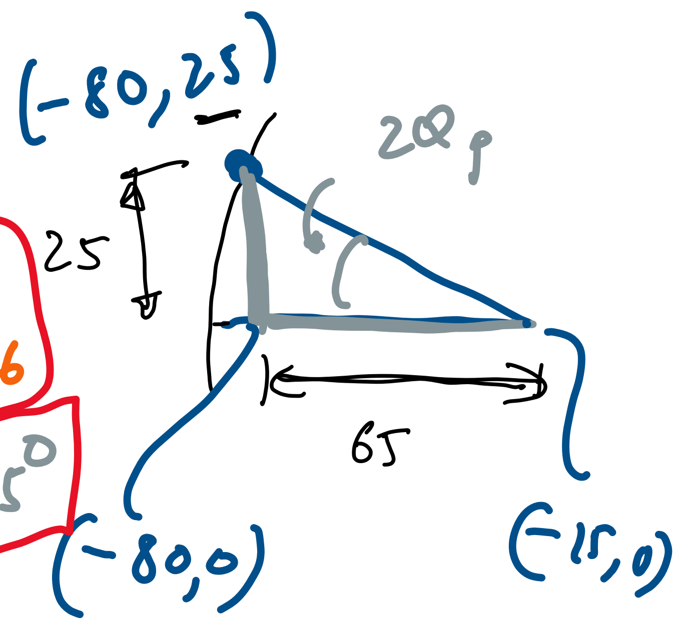
$$\text{center} = \left(\frac{-80+50}{2}, 0\right)$$
$$= (-15, 0)$$

②



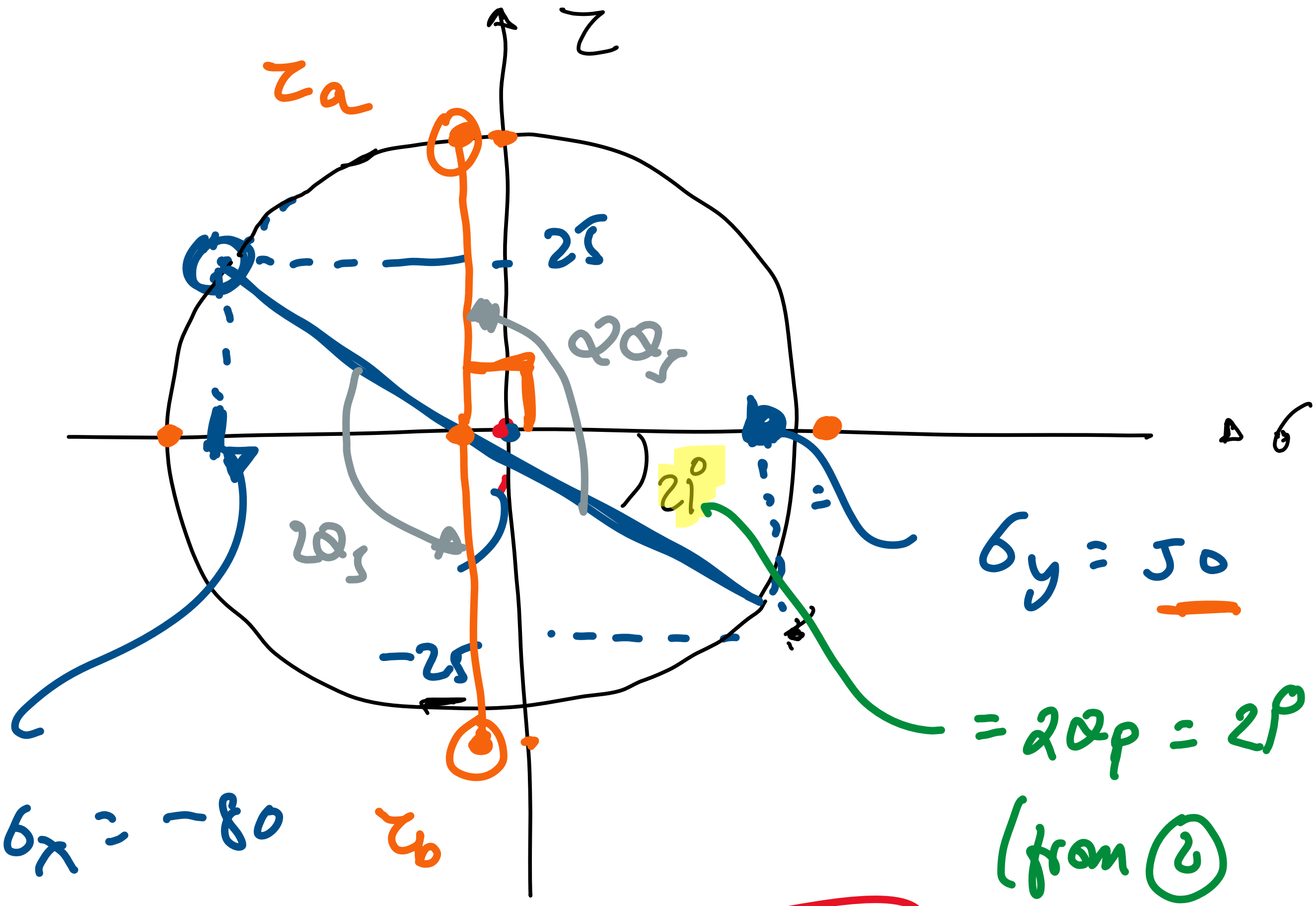
$$r = \sqrt{(50-(-15))^2 + (-25-0)^2}$$

$$= 69.6$$



$$\boxed{\begin{aligned} \sigma_a &= 69.6 - 15 = 54.6 \\ \sigma_b &= -69.6 - 15 = -84.6 \end{aligned}}$$

$$\tan 2\theta_p = \frac{25}{65} \;;\; \boxed{\theta_p = 10.5^\circ}$$



$$\boxed{\tau_a = -\tau_b = r = 69.6\ \text{MPa}}$$

$$2\theta_s = 90 + 21^\circ = 111^\circ$$

$$\boxed{\theta_s = 55.5^\circ}$$

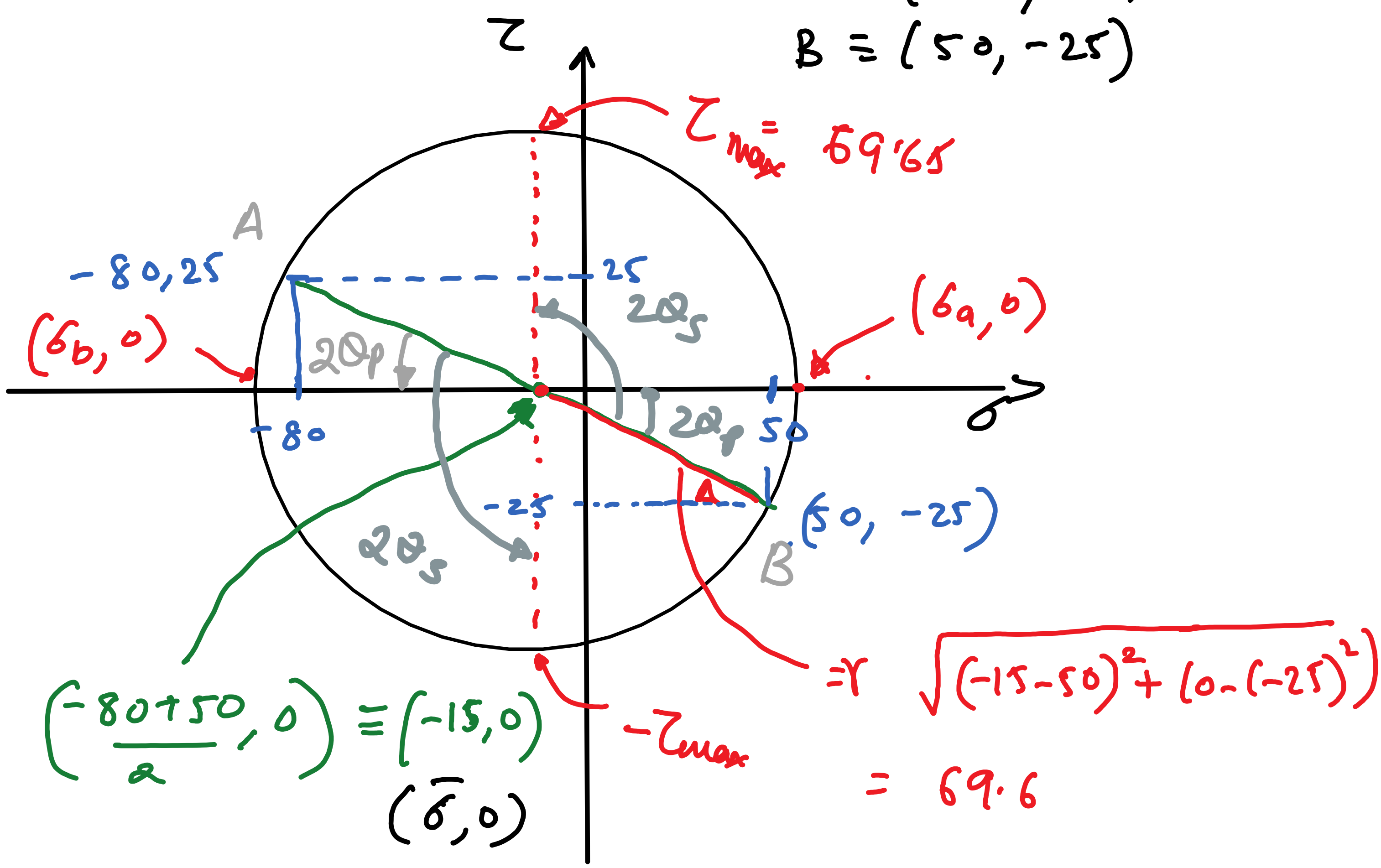
Redo 1, 2, 3 using formula

V, VI   III, III*   IV, IV*

Here is a better drawing of the Mohr's circle

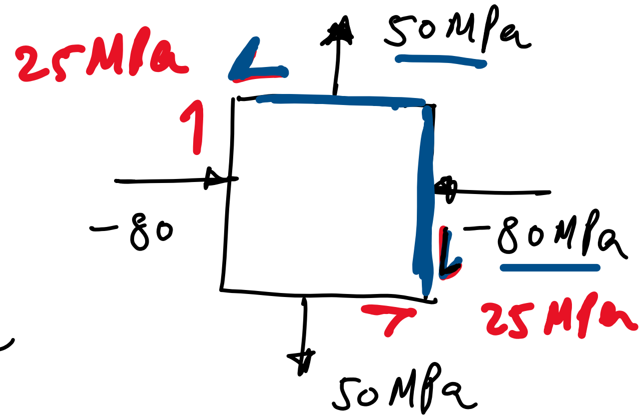
$A \equiv (-80, 25)$

$B = (50, -25)$



CW + / CCW −

EXAMPLE 2



① Compute the stress for a 30° Clockwise rotation of the plane stress element

② Draw the corresponding stress element and show the stresses and their proper direction.

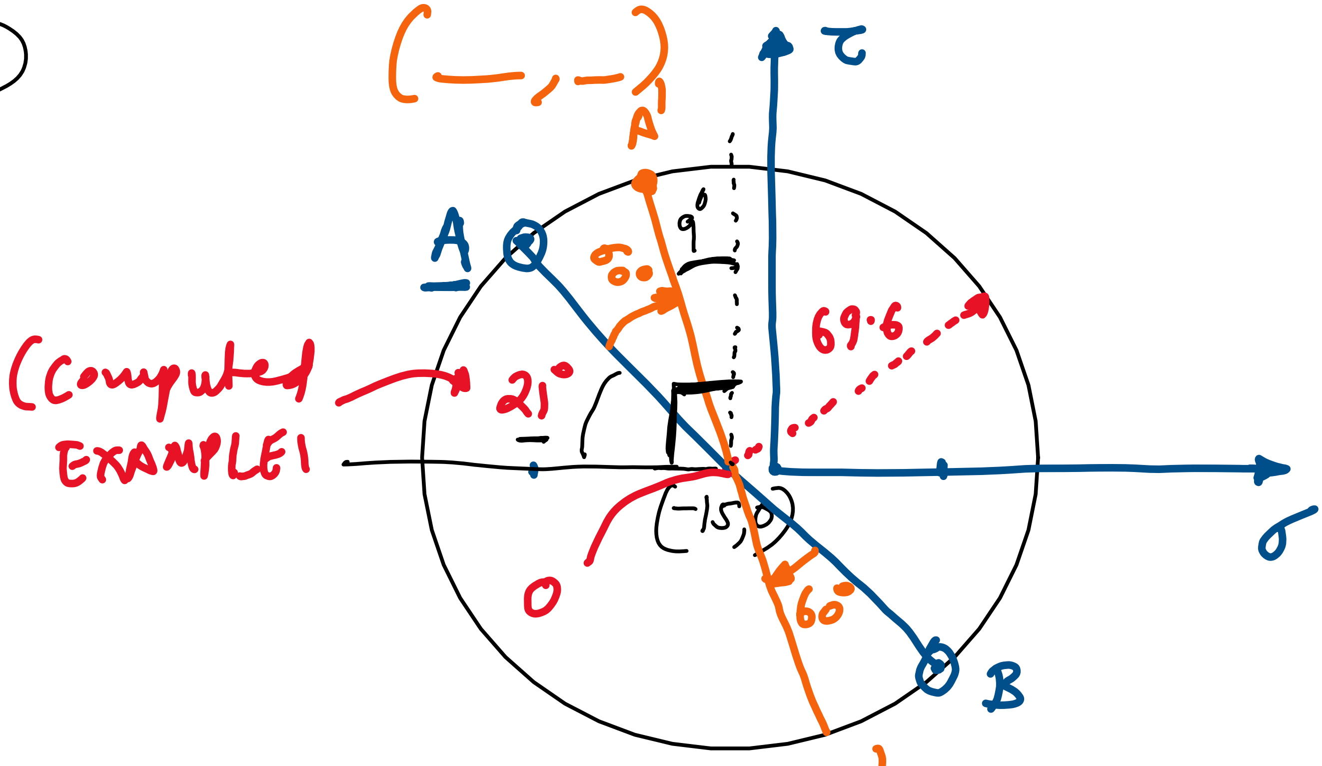
Using Mohr's circle

$$(\sigma_c, 0) = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{-80+50}{2}, 0\right) = (-15, 0)$$

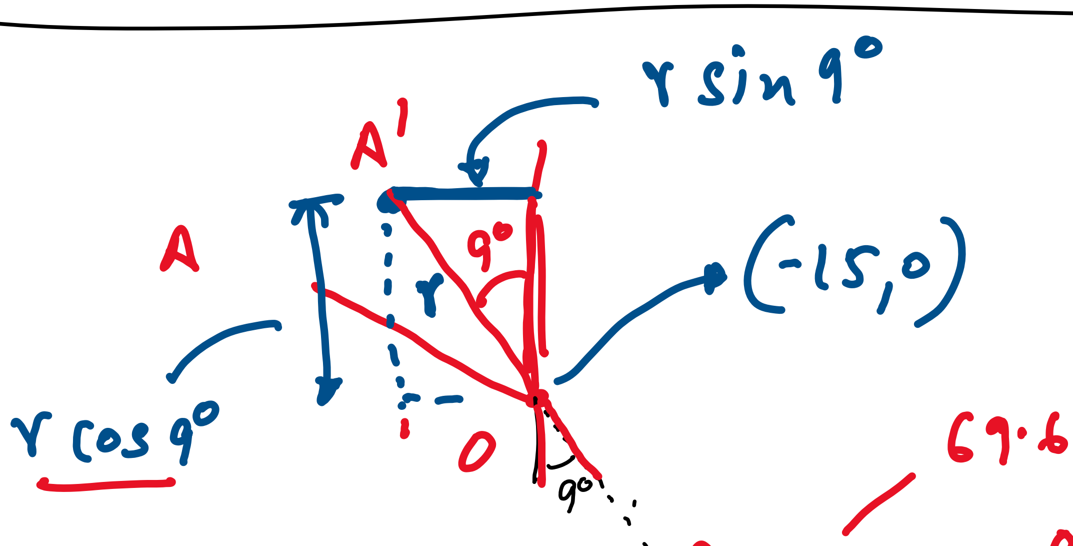
$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-80-50}{2}\right)^2 + (-25)^2}$$

$$= \underline{69.6}$$

1



Face A: −80, +25 (A)
Face B: 50, −25 (B)



$A' \equiv (-15 - r\sin 9^\circ,\ r\cos 9^\circ)$

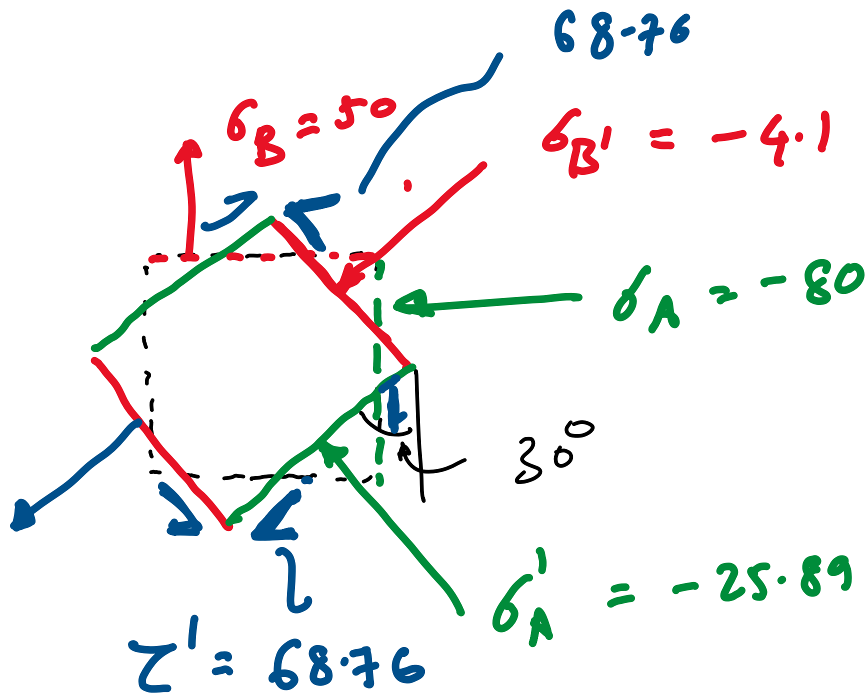
$A' \equiv (-25.89,\ 68.76)$

$B' \equiv (-4.1,\ -68.76)$

Check this yourself

$B' \equiv (-4.1, -68.76)$ Check this yourself using trig.

②



Another way without using the Mohr's circle

(I) $$\sigma = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta \quad \checkmark$$

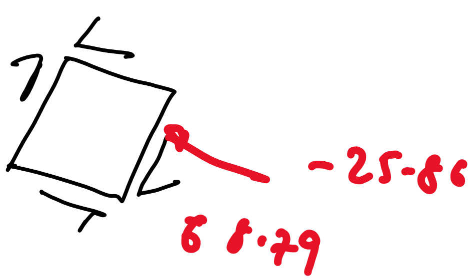
(II) $$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy} \cos 2\theta \quad \checkmark$$

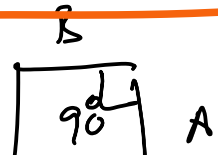
Stresses on face A'

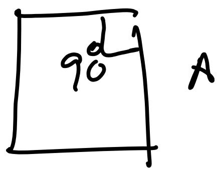
$\theta = \underline{-30°}$ (CW) ; $\sigma_x = -80$; $\sigma_y = 50$; $\tau_{xy} = -25$ MPa

NOTE: see the first page

$$\boxed{\begin{aligned} \sigma_{A'} &= -25.85 \\ \tau_{A'} &= -68.79 \end{aligned}}$$






90°
A

For face B'. B' is at 90° to face A'

$\theta = 90 - 30 = 60°$

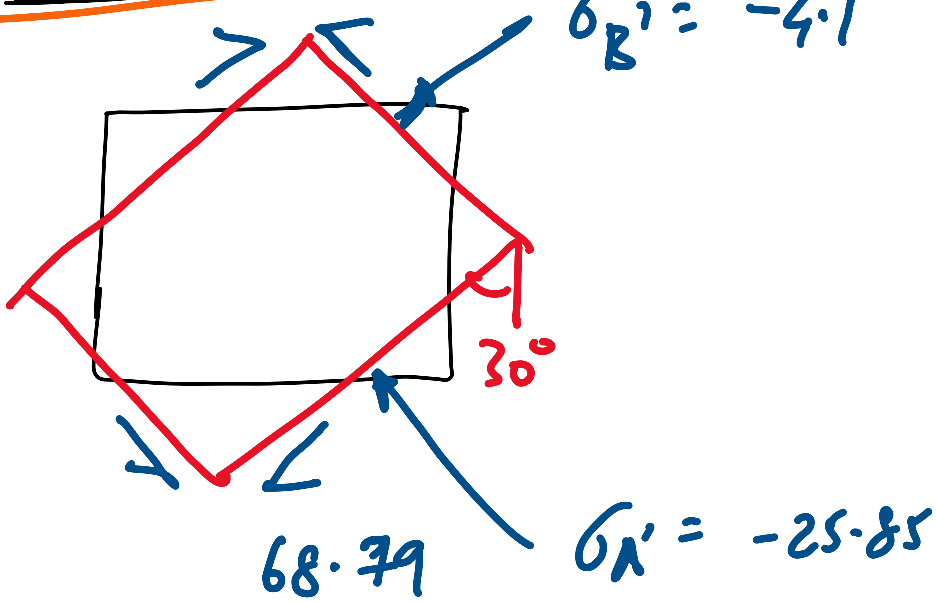
$\sigma_x = -80$; $\sigma_y = 50$; $\tau_{xy} = -25$

Use formula (I) & (II)

$\sigma_{B'} = -4.1$ MPa

$\tau_{B'} = 68.79$ MPa

(2)



EXAM : 1

Feb 6th 9:30 – 10:45 AM

3 problems ≃ 30 or 25

HW 1 – 3 (Lecture 1 – 6)

Mock exam (Bonus points : 3 points to be added to EXAM 1)