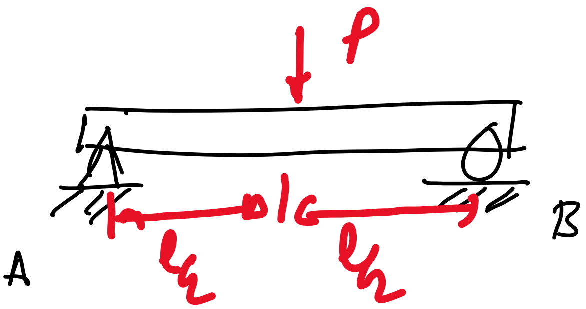
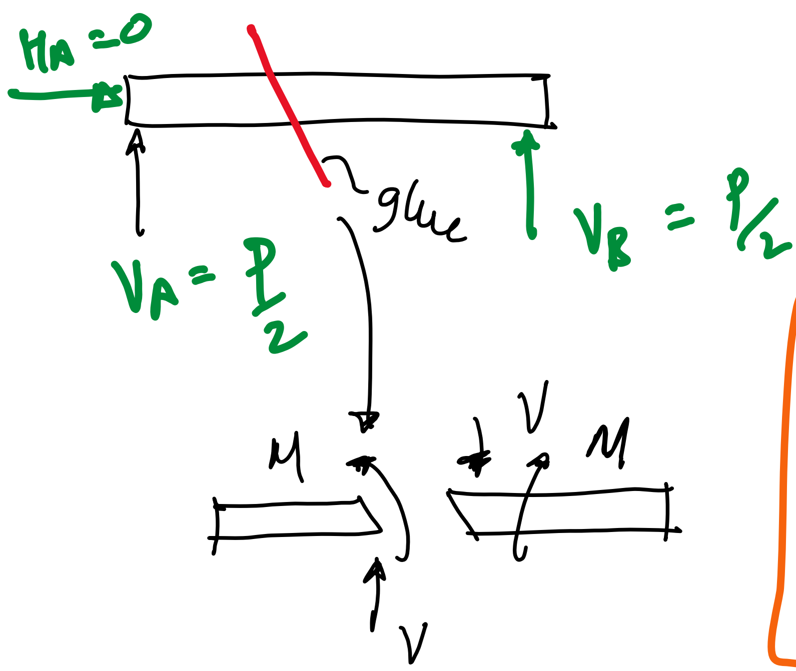
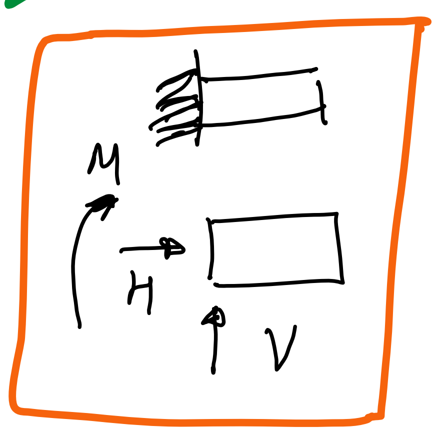
Shear Force and Bending Moment Diagrams

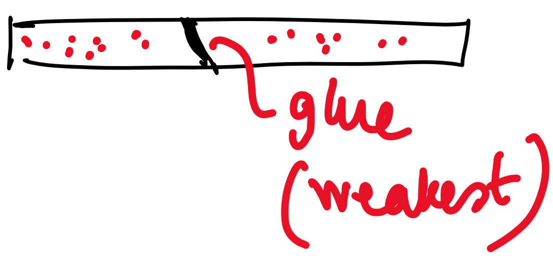
Concept



FBD







This section: $\underline{V, M}$

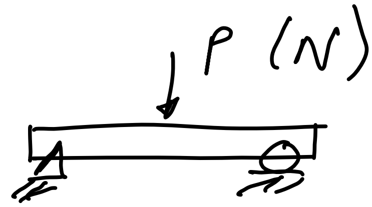
Next section: $\underline{\text{Connecting}}$ V, M $\rightarrow$ σ, τ

σ: normal stress

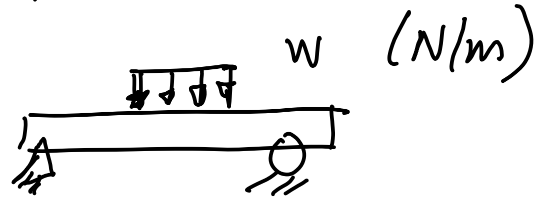
τ: shear stress

Types of forces

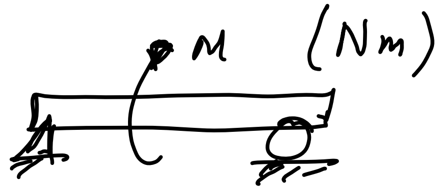
(1) Concentrated load



(2) Distributed load



(3) Moment



Formula

q – load intensity
V – shear force
M – bending moment

$$\boxed{V = \frac{dM}{dx}} \qquad \boxed{q = \frac{dV}{dx} = \frac{d^2M}{dx^2}}$$

$$\int_A^B dM = \int_A^B V\,dx$$

$$\boxed{M_B - M_A = \int_A^B V\,dx}$$

$$\boxed{V_B - V_A = \int_A^B q\,dx}$$

Draw shear force / Bending moment diagram

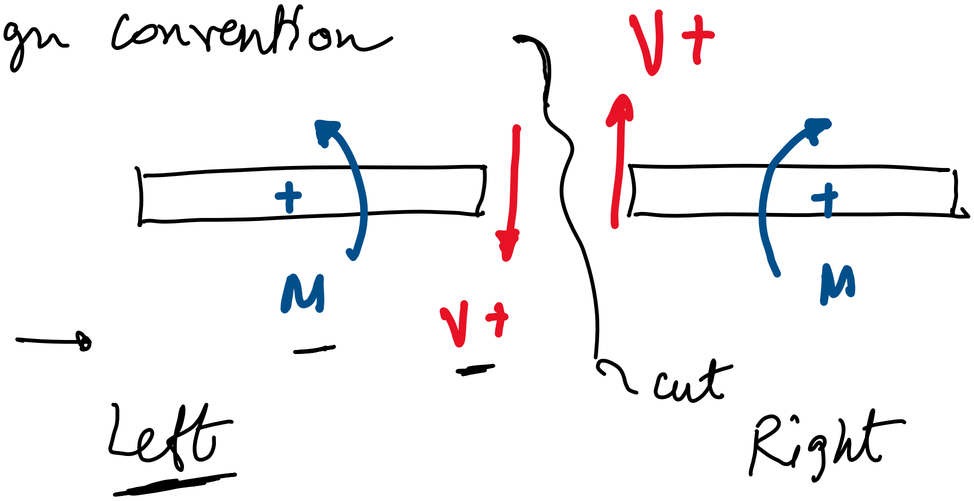
1. Draw a FBD
2. Assign a co-ordinate frame
3. Compute the reaction forces

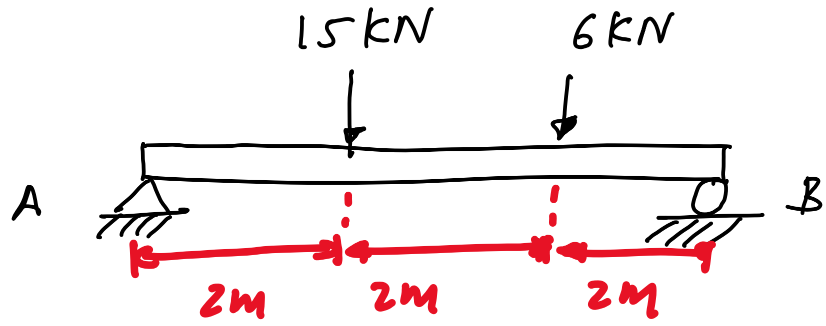
$$\sum F_x = 0 \; ; \quad \sum F_y = 0 \; ; \quad \sum M_z = 0$$

4. Compute the shear force / bending moment at every location of the beam using these equation

$$V = \frac{dM}{dx} \; ; \quad q = \frac{dV}{dx}$$

Sign convention





Draw the shear force and bending moment diagram for the beam shown above.

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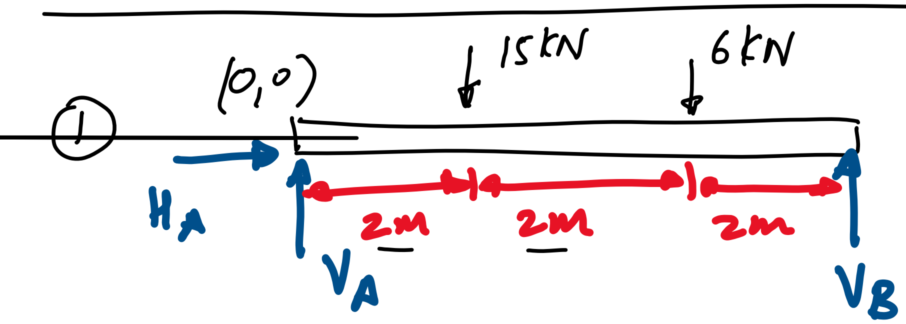
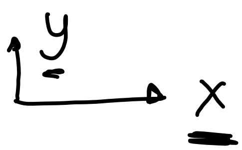
① 

② 

③ $\Sigma F_x = 0 \Rightarrow H_A = 0$ — (I)

$\Sigma F_y = 0 \Rightarrow V_A - 15 - 6 + V_B = 0$ — (II)

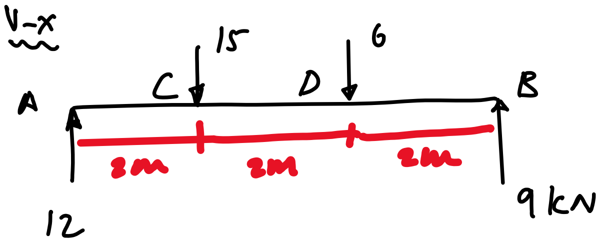
$\Sigma M_A = 0 \Rightarrow -15(2) - 6(4) + V_B(6) = 0$ — (III)

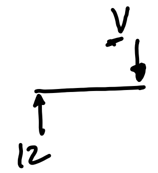
$V_B = 9\ kN$

Substitute $V_B = 9\ kN$ in (II)

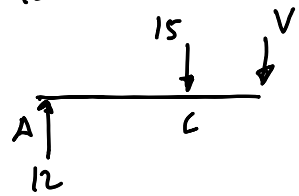
$V_A = 12\ kN$

V-x

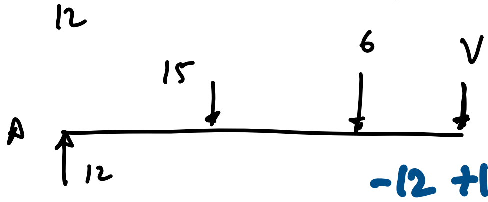




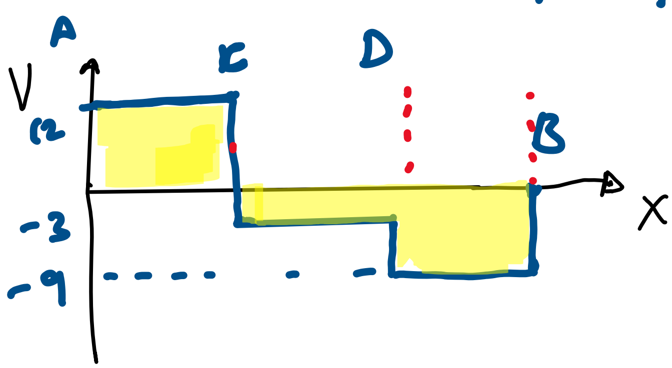
$$+V-12=0$$
$$V=12$$



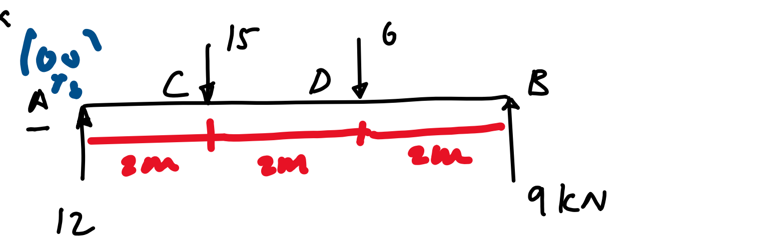
$$+V-12+15=0$$
$$V=-3$$

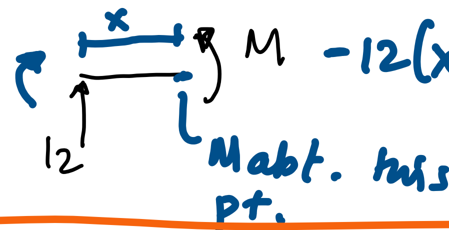


$$-12+15+6+V=0$$
$$V=-9$$

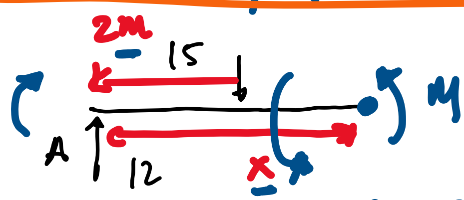


M-x

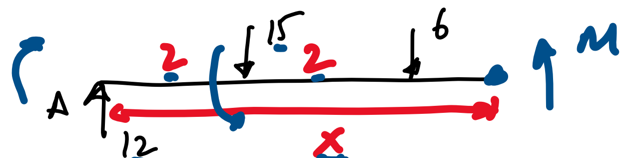




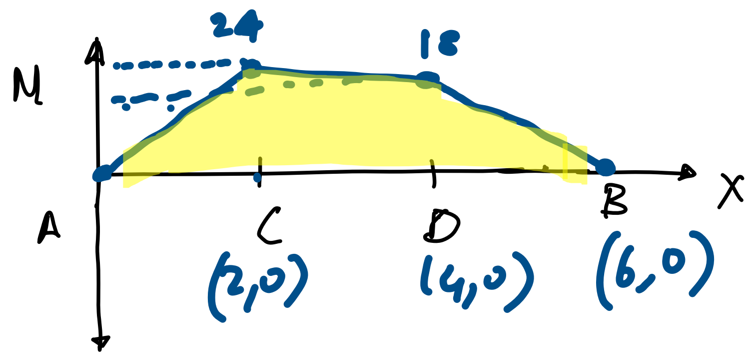
$$-12(x)+M=0$$
$$\boxed{M=12x}$$



$$-12(x)+15(x-2)+M=0$$
$$\boxed{M=-3x+30} \quad 2\leq x\leq 4$$



$$-12(x)+15(x-2)+6(x-4)+M=0$$
$$\boxed{M=-9x+54} \quad 4\leq x\leq 6$$



Bending Moment Diagram

Another method to draw the SFD & BMD

Shear force diagram

$$q = \frac{dV}{dx} \qquad V = \frac{dM}{dx}$$

If $q = 0$

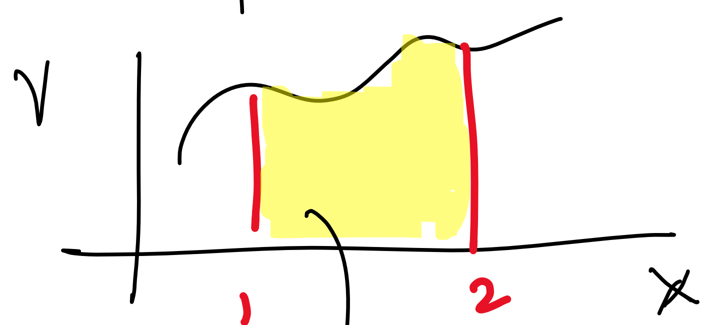
$\int dV = 0 \Rightarrow V_2 - V_1 = 0$

same as before

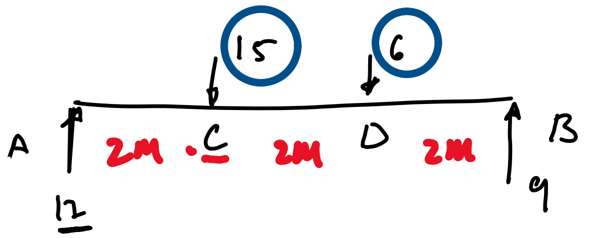
$$\int_1^2 dM = \int_1^2 V\,dx$$

$$M_2 - M_1 = \int_1^2 V\,dx$$

area under V-x curve between point 1 & 2



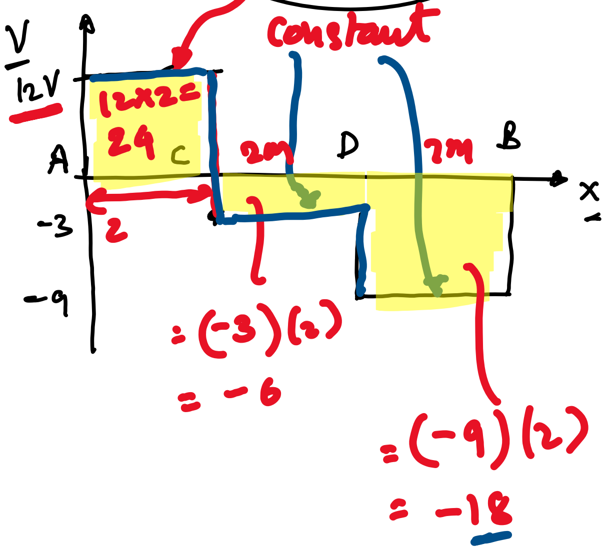
$$\int_1^2 V\,dx = M_2 - M_1$$

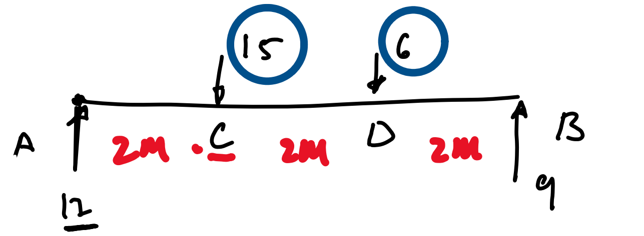


$q = 0$

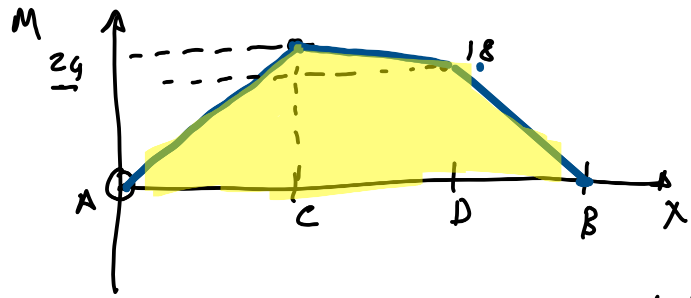
$V_2 - V_1 = 0$

constant





$M_2 - M_1 = \int_1^2 V\,dx$ = area under V-x curve between 1 and 2



$M_C - M_A$ = area under V-x between A and C

$= 24$

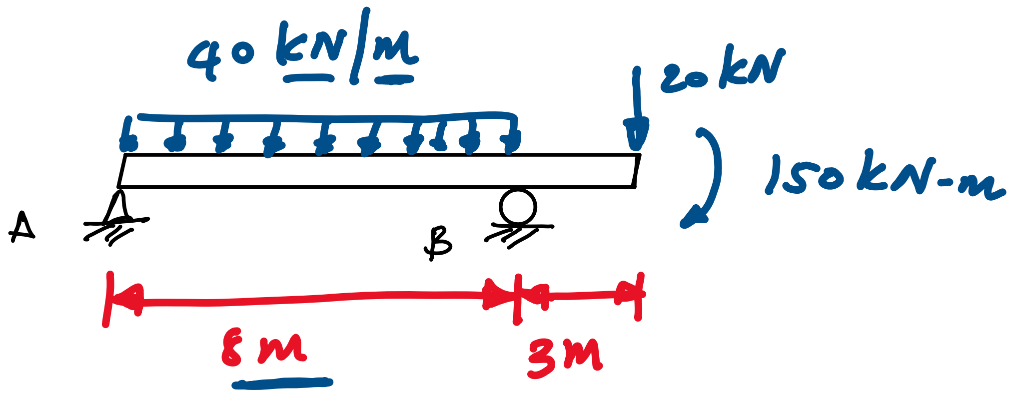
$M_C = M_A + 24 = 0 + 24 = 24$

$M_C - M_A = \int V\,dx$

$= \int c\,dx = cx$

$M_D - M_C = \int_C^D V\,dx = -6$

$M_D - 24 = -6 \Rightarrow M_D = 18$



Draw the shear force and bending moment diagram

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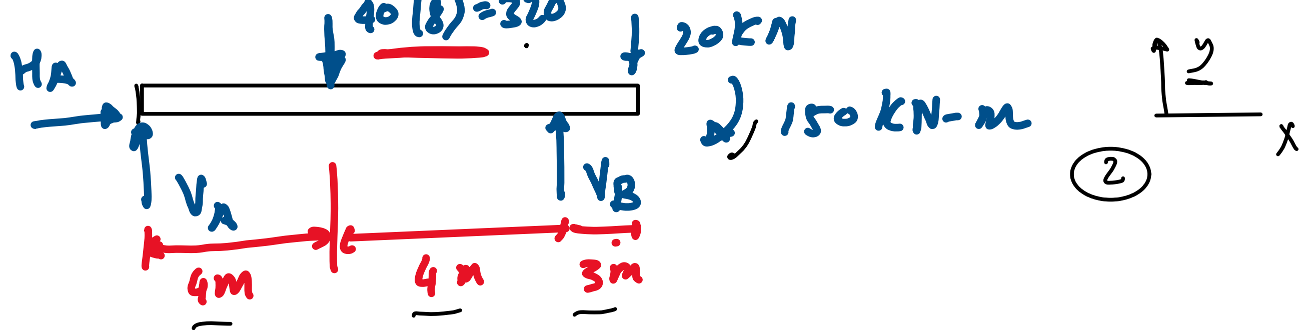
① 

③ $\Sigma F_x = 0 \quad \Rightarrow \quad H_A = 0 \quad$ — I

$\Sigma F_y = 0 \quad \Rightarrow \quad V_A - 320 - 20 + V_B = 0$

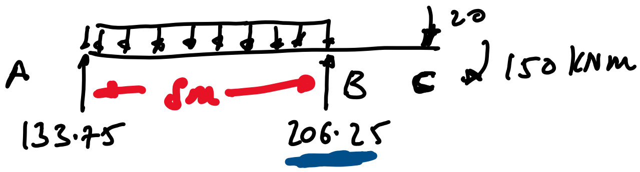
$V_A + V_B = 340 \quad$ — II

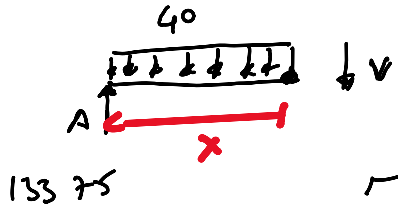
$\Sigma M_A = 0 \quad \Rightarrow \quad -(320)(4) + V_B(8) - 20(11) - 150 = 0$

$V_B = 206.25\ kN \quad$ — III

Substitute $V_B = 206.25$ in II & solve for V_A

$V_A = 133.75\ kN$

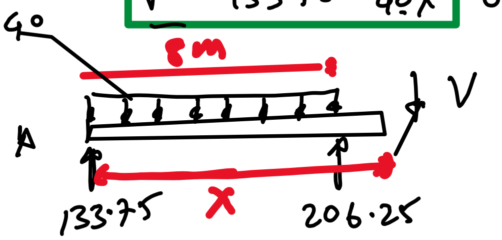




net force

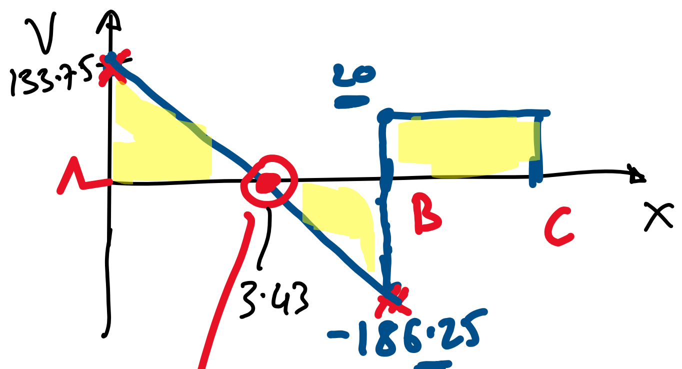
$+V - 133.75 + (40)x = 0$

$\boxed{V = 133.75 - 40x}$ $\quad 0 \le x \le 8$



$+V - 133.75 - 206.25 + 40(8) = 0$

$\boxed{V = 20}$ $\quad 8 \le x \le 11$

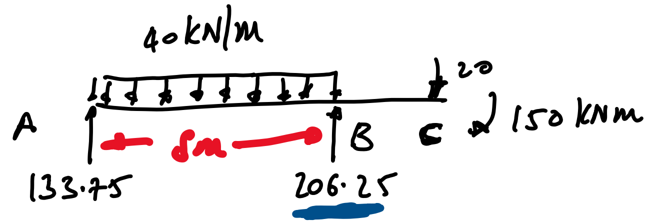


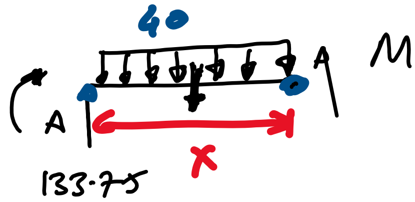
$= 133.75 - 40(8)$

when does $V=0$

$0 = 133.75 - 40x$

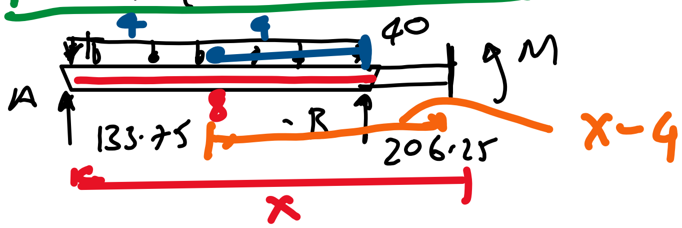
$x = 3.43$





$-133.75(x) + \underbrace{40x}_{\text{net force}}\underbrace{(x/2)}_{\text{moment}} + M = 0$

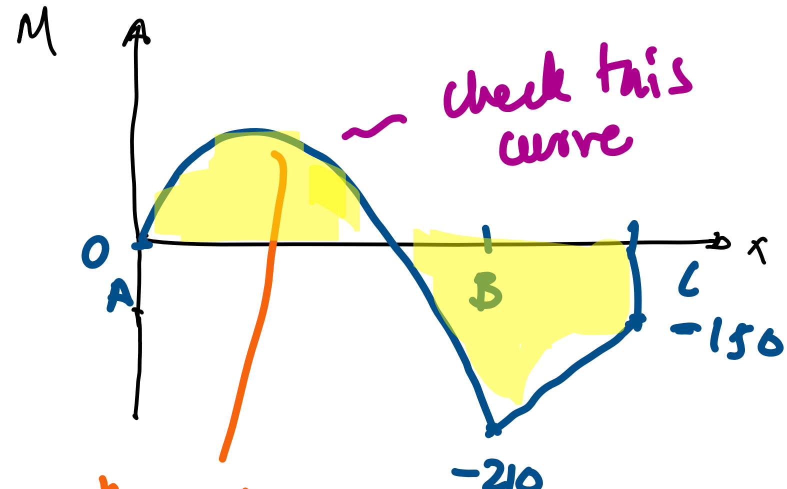
$\boxed{M = x(133.75 - 20x)}$ $\quad 0 \le x \le 8$



$-(133.75)(x) + 40(8)(x-4)$

$- 206.25(x-8) + M = 0$

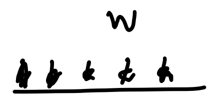
$\boxed{M = 20x - 370}$ $\quad 8 \le x \le 11$



need to subs some intermediate value

intermediate value

Solving in another way



$$\frac{dV}{dx} = q = \underbrace{-w}_{\text{constant}}$$

$$\int_1^2 dV = \int_1^2 -w\,dx$$

$$V_2 - V_1 = -w(x_2 - x_1)$$

$V \propto x$ (linear)

shear force ... distance

$$\frac{dM}{dx} = V$$

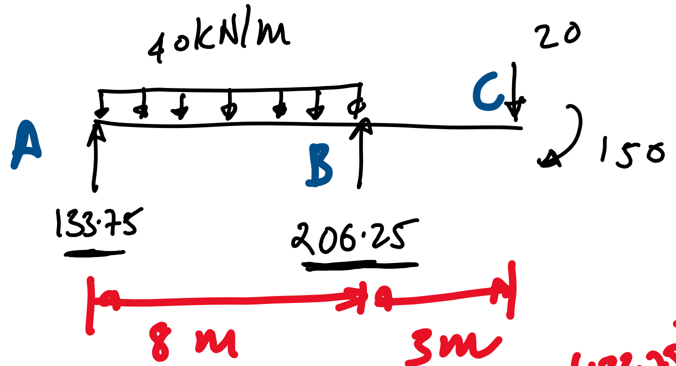
$$\int_1^2 dM = \int_1^2 V\,dx$$

$$M_2 - M_1 = \int_1^2 (-wx)\,dx \qquad \{\text{Assume } V = -wx\}$$

$$= -w\left(\frac{x_2^2}{2} - \frac{x_1^2}{2}\right)$$

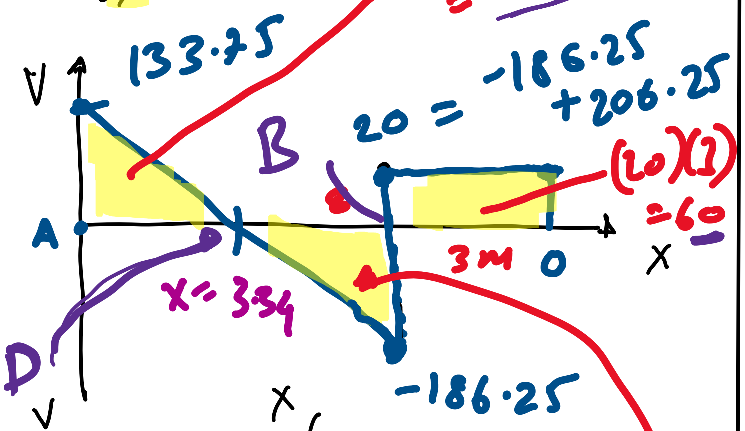
$M \propto x^2$ (quadratic)

moment $\propto$ distance^2



$\frac{dV}{dx} = -W$

$(\frac{1}{2})(3.34)(133.75)$
$= 223.36$



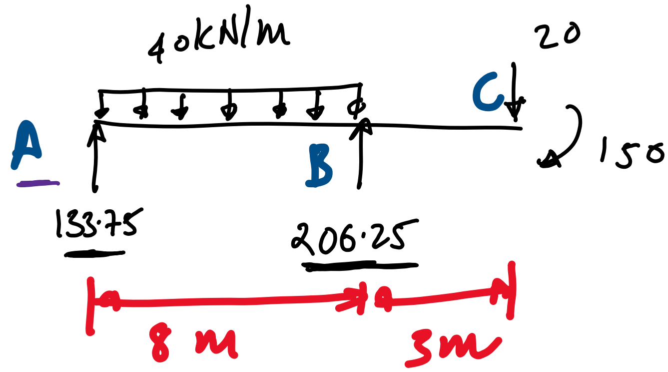
$\int_{133.75}^{V} dV = -40 \int_0^x dx$

$V - 133.75 = -40\,x$ $\quad \boxed{0 \le x \le 8}$

$x = 8 \quad V_B = -186.25$

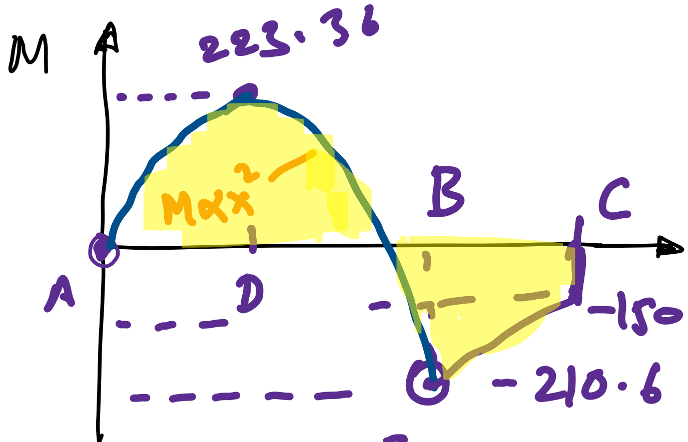
$0 - 133.75 = -40(x)$
$x = 3.34$

$\frac{1}{2}(8-3.34)(-186.25)$
$= -433.36$
check



$\frac{dM}{dx} = V \qquad M_2 - M_1 = \int_1^2 V dx$

area under V-x curve from 1-2



$M_B - M_D = \int_D^B V dx$

$M_B - 223.36 = -433.36$

$M_B = -210.\cancel{6}$
-210

$M \propto \int V dx$
$\quad wx$

$M \propto \int x dx = \frac{x^2}{2}$

$M_C - M_B = 60$

$M_C - (-210) = 60 \Rightarrow M_C = 150$

$M_c - (-210) = 60 \Rightarrow M_c = 150$