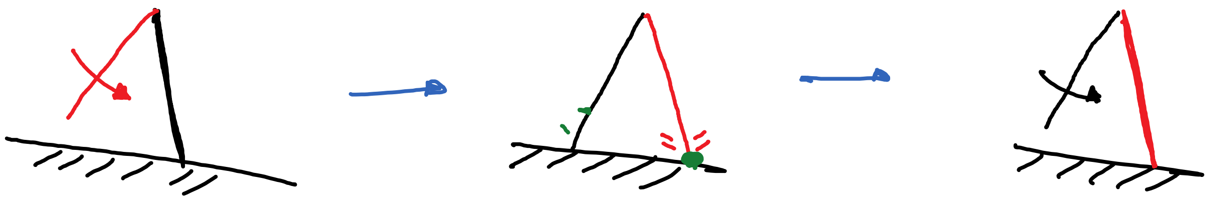
Passive dynamic walker



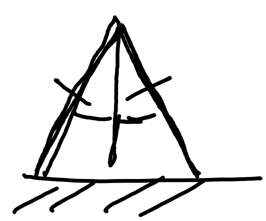
→ (a) single stance $\xrightarrow[\text{ground}]{\text{(b) detect}}$ foot-strike (c) $\xrightarrow{\otimes}$ single stance → footstrike

repeating unit

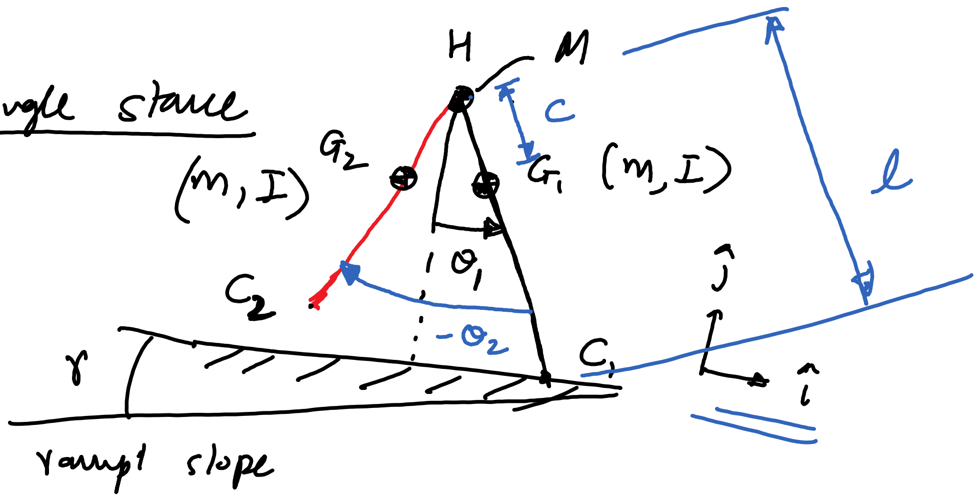
(plastic collision)

Equations for

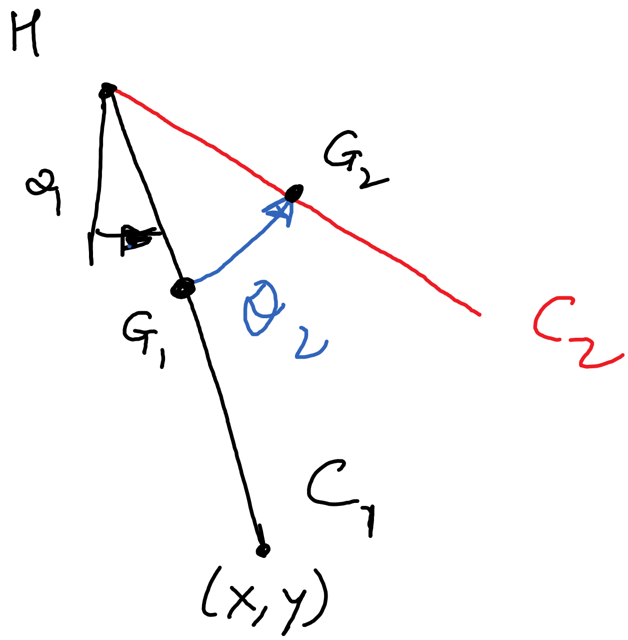
1. single stance
2. footstrike
3. detect ground →



① Single stance



$\theta_1 \rightarrow$ absolute angle



Floating Base $\rightarrow (\underline{\underline{x, y}}, \theta_1, \theta_2)$

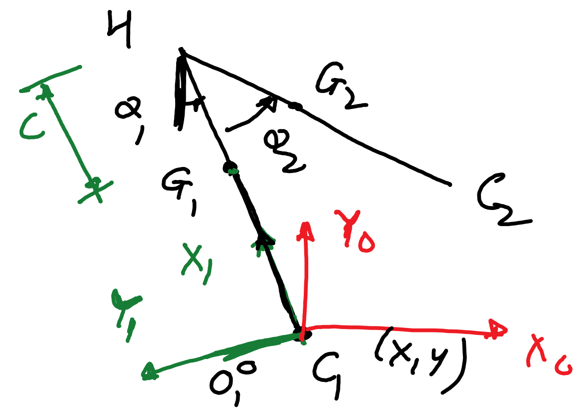
Euler-Lagrange to derive the equation

① positions / velocity

② $\mathcal{L} = T - V$

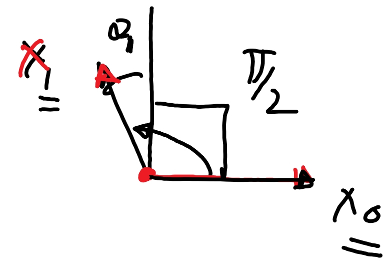
③ $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = Q_j$

① $r_G^0 = \begin{pmatrix} x \\ y \end{pmatrix}$



$$\begin{pmatrix} r_{G_1}^0 \\ \underline{1} \end{pmatrix} = \begin{pmatrix} R_1^0 & O_1^0 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} l - c \\ 0 \\ 1 \end{bmatrix}.$$

$$O_1^0 = \begin{pmatrix} x \\ y \end{pmatrix} \qquad R_1^0 = \begin{pmatrix} C_1 & -S_1 \\ S_1 & C_1 \end{pmatrix}$$

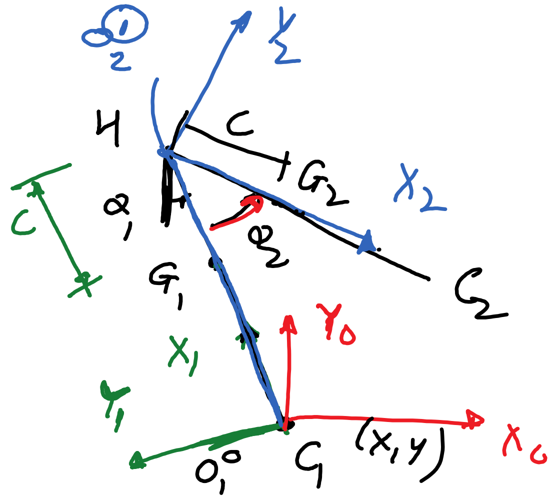


$$C_1 = \cos\left(\tfrac{\pi}{2} + Q_1\right)$$
$$S_1 = \sin\left(\tfrac{\pi}{2} + Q_1\right)$$

$$\begin{pmatrix} r_H \\ 1 \end{pmatrix} = \begin{pmatrix} R_1^0 & O_1^0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} l \\ 0 \\ 1 \end{pmatrix}$$

$$r^1_{G_2} = (H_0^1)(H_1^2)\begin{pmatrix} c \\ 0 \\ 1 \end{pmatrix}$$

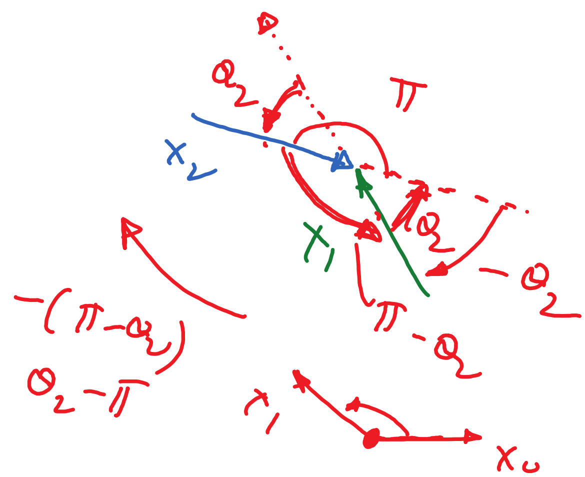
$$r^1_{G_2} = \begin{pmatrix} R_1^0 & O_1^0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} R_2^1 & O_2^1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} c \\ 0 \\ 1 \end{pmatrix}$$



$$O_2^1 = \begin{bmatrix} l \\ 0 \end{bmatrix} \begin{matrix} \leftarrow \text{along } X_1 \\ \leftarrow \text{along } Y_1 \end{matrix}$$

$$R_2^1 = \begin{pmatrix} c_2 & -s_2 \\ s_2 & c_2 \end{pmatrix} \qquad \underline{\underline{\theta_2 - \pi}}$$

$$c_2 = \cos(\theta_2 - \pi)$$
$$s_2 = \sin(\theta_2 - \pi)$$



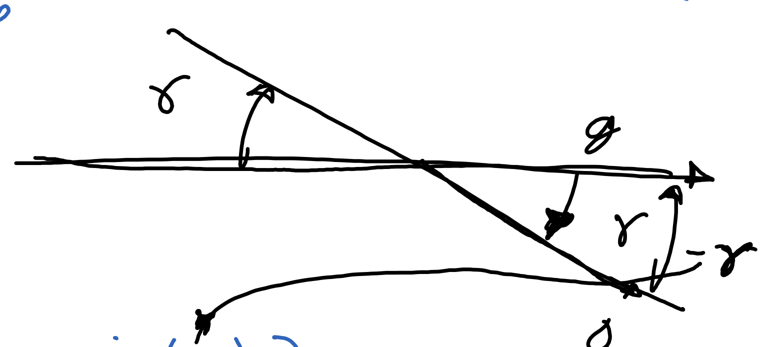
$$\begin{pmatrix} r^0_{C_2} \\ 1 \end{pmatrix} = H_1^0 H_2^1 \begin{pmatrix} l \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} R_1^0 & O_1^0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} R_2^1 & O_2^1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} l \\ 0 \\ 1 \end{pmatrix}$$

---

g — ground, $0 \rightarrow$ ramp

$$\begin{cases} r^g_{G_1} = R\, r^0_{G_1} \\ r^g_{G_2} = R\, r^0_{G_2} \\ r^g_H = R\, r^0_H \end{cases}$$



$$R = \begin{pmatrix} \cos(-\gamma) & -\sin(-\gamma) \\ \sin(-\gamma) & \cos(-\gamma) \end{pmatrix}$$

$$r^g_{G_1} = \begin{pmatrix} x^g_{G_1} \\ y^g_{G_1} \end{pmatrix} \quad r^g_{G_2} = \begin{pmatrix} x^g_{G_2} \\ y^g_{G_2} \end{pmatrix} \quad r^g_H = \begin{pmatrix} x^g_H \\ y^g_H \end{pmatrix}$$

$$V_{G_1} = \dot{r}^o_{G_1} = \begin{pmatrix} \dot{x}^o_{G_1} \\ \dot{y}^o_{G_1} \end{pmatrix} \quad \text{and } \underline{\underline{\text{so on}}}$$

$$\underline{\underline{\partial r}}$$

$$= J_{G_1} \dot{q} = \underset{2\times 4}{J_{G_1}} \underset{4\times 1}{\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}}$$

$$\begin{pmatrix} \dfrac{\partial x^o_{G_1}}{\partial x} & \dfrac{\partial x^o_{G_1}}{\partial y} & \dfrac{\partial x^o_{G_1}}{\partial \theta_1} & \dfrac{\partial x^o_{G_1}}{\partial \theta_2} \\ \dfrac{\partial y^o_{G_1}}{\partial x} & \dfrac{\partial y^o_{G_1}}{\partial y} & \dfrac{\partial y^o_{G_1}}{\partial \theta_1} & \dfrac{\partial y^o_{G_1}}{\partial \theta_2} \end{pmatrix}$$

② $\mathcal{L} = T - \underline{V} = \frac{1}{2} m V_{G_1}^T V_{G_1} + \frac{1}{2} m V_{G_2}^T V_{G_2} + \frac{1}{2} M V_H^T V_H$

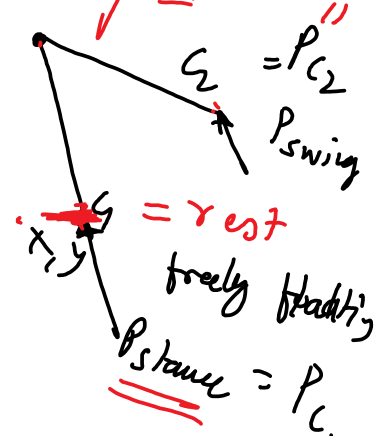
$$\frac{1}{2} I \dot{\theta}_1^2 + \frac{1}{2} I (\dot{\theta}_1 + \dot{\theta}_2)^2$$

stance   swing

$$-\left(\underline{M} g y_H^g + mg V_{G_1}^g + mg V_{G_2}^g\right)$$

③ E-L

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = \underline{Q_j} = \underbrace{J_{C_1}^T}_{\text{jacobian \& static forces}} P_{C_1} + \cancel{J_{C_2}^T P_{C_2}}^{\,0}$$



$$\begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \end{pmatrix}_{2\times 4}$$

$$M(q)\,\ddot{q} = B(q,\dot{q}) + \underset{4\times 2}{J_{C_1}^T}\ \underset{2\times 1}{P_{C_1}}$$

$$q = \begin{pmatrix} x \\ y \\ \theta_1 \\ \theta_2 \end{pmatrix}$$

$$\begin{array}{l} ① \\ ② \\ \\ \\ \end{array}\begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} \ddot{x} = 0 \\ \ddot{y} = 0 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix} + \begin{pmatrix} J_{11} & J_{21} \\ J_{12} & J_{22} \\ J_{13} & J_{23} \\ J_{14} & J_{24} \end{pmatrix} \begin{pmatrix} P_{C_1}^x \\ P_{C_1}^y \end{pmatrix} \neq 0$$

① & ② gives $P_{C_1}^x$ & $P_{C_1}^y$

③ & ④

$$
\begin{array}{c}
① \\ ② \\ ③ \\ ④
\end{array}
\begin{pmatrix}
M_{11} & M_{12} & M_{13} & M_{14} \\
M_{21} & M_{22} & M_{23} & M_{24} \\
M_{31} & M_{32} & M_{33} & M_{34} \\
M_{41} & M_{42} & M_{43} & M_{44}
\end{pmatrix}
\begin{pmatrix}
\ddot{x} = 0 \\ \ddot{y} = 0 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2
\end{pmatrix}
=
\begin{pmatrix}
B_1 \\ B_2 \\ B_3 \\ B_4
\end{pmatrix}
+
\underbrace{\begin{pmatrix}
J_{11} & J_{21} \\
J_{12} & J_{22} \\
J_{13} & J_{23} \\
J_{14} & J_{24}
\end{pmatrix}}_{4\times 2}
\underbrace{\begin{pmatrix}
P_{c_1}^x \\ P_{c_1}^y
\end{pmatrix}}_{2\times 1}
\neq 0
$$

① & ② gives $P_{c_1}^x$ & $P_{c_1}^y$

$$
\begin{aligned}
(i)\quad & M_{13}\ddot{\theta}_1 + M_{14}\ddot{\theta}_2 = B_1 + J_{11} P_{c_1}^x + \cancel{J_{21}} P_{c_1}^y \\
(ii)\quad & M_{23}\ddot{\theta}_1 + M_{24}\ddot{\theta}_2 = B_2 + \cancel{J_{12}} P_{c_1}^x + J_{22} P_{c_1}^y \\
(iii)\quad & \boxed{M_{33}\ddot{\theta}_1 + M_{34}\ddot{\theta}_2 = B_3} + \cancel{J_{13}} P_{c_1}^x + \cancel{J_{23}} P_{c_1}^y \\
(iv)\quad & \boxed{M_{43}\ddot{\theta}_1 + M_{44}\ddot{\theta}_2 = B_4} + \cancel{J_{14}} P_{c_1}^x + \cancel{J_{24}} P_{c_1}^y
\end{aligned}
$$

4 eq & 4 unkn

$$
J_{c_1} = \left(\frac{\partial\, x_{c_1}\ y_{c_1}}{\partial(x, y, \theta_1, \theta_2)}\right)_{2\times 4}
$$

$$
= \begin{pmatrix}
\frac{\partial x}{\partial x} & \cancel{\frac{\partial x}{\partial y}} & \cancel{\frac{\partial x}{\partial \theta_1}} & \cancel{\frac{\partial x}{\partial \theta_2}} \\
\cancel{\frac{\partial y}{\partial x}} & \frac{\partial y}{\partial y} & \cancel{\frac{\partial y}{\partial \theta_1}} & \cancel{\frac{\partial y}{\partial \theta_2}}
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix}
$$

$c_1\ (x, y)$

Use (iii) & (iv) to solve for $\ddot{\theta}_1, \ddot{\theta}_2$

$$\left.\begin{aligned} M_{33}\,\ddot{\theta}_1 + M_{34}\,\ddot{\theta}_2 &= B_1 \\ M_{43}\,\ddot{\theta}_1 + M_{44}\,\ddot{\theta}_2 &= B_2 \end{aligned}\right\} \text{ solve for } \ddot{\theta}_1, \ddot{\theta}_2$$

Use $\ddot{\theta}_1, \ddot{\theta}_2$ to solve for $P_{c_1}^x, P_{c_1}^y$

$$\left.\begin{aligned} M_{13}\,\ddot{\theta}_1 + M_{14}\,\ddot{\theta}_2 &= B_1 + P_{c_1}^x \\ M_{23}\,\ddot{\theta}_1 + M_{24}\,\ddot{\theta}_2 &= B_2 + P_{c_1}^y \end{aligned}\right\}$$