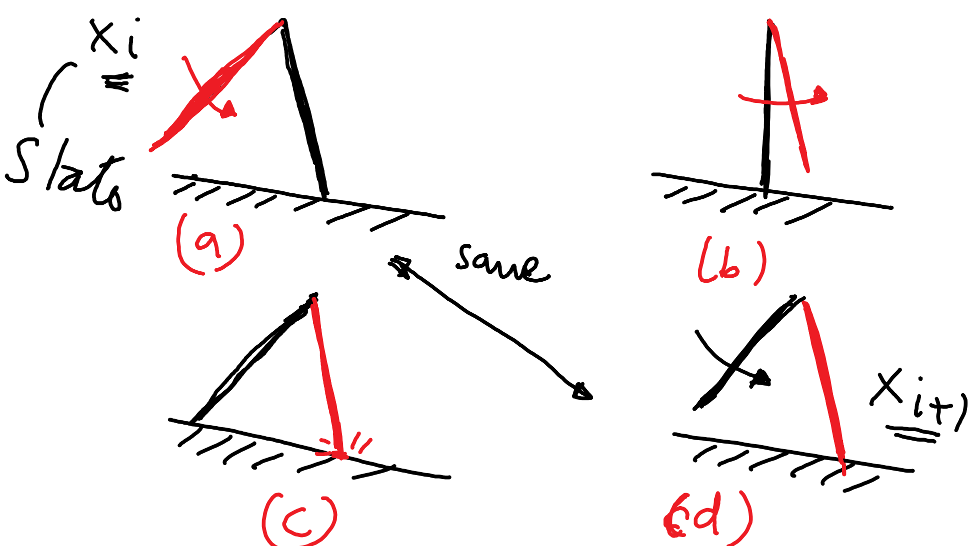
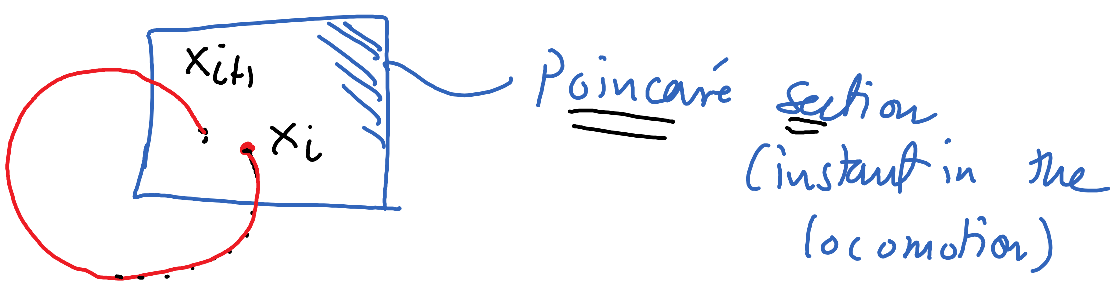
Introduction to passive dynamic walking

Passive dynamic walker is a machine that walks down a ramp without any external power or energy.



Tools from dynamical systems.

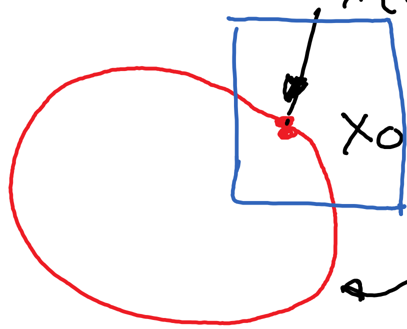
Poincaré section & map



$$x_{i+1} = F(x_i)$$

Poincaré map

$x_{i+1} = x_0$ — fixed point



$x_0 = x_i$

limit cycle.

$$\boxed{x_0 = F(x_0)}$$

x_0 - periodic

Stability (linearized)

$x_{i+1} = F(x_0 + \Delta x)$

$x_i = x_0 + \Delta x$ where Δx is small

$x_{i+1} = ?$ going away from x_0 (unstable)

going toward x_0 (stable)

$$\boxed{J = \left.\frac{\partial F}{\partial x}\right|_{x = x_0}}$$

poincare map wrt. state

$|\max(\operatorname{eig} J)| > 1$ unstable

$|\max(\operatorname{eig} J)| < 1$ stable

$|\max(\operatorname{eig} J)| = 1$ neutrally stable.

Example : consider

$$x_{i+1} = F(x_i) = x_i^2$$

Find the fixed points and their stability

Intuition

$0 < x_i \leq 1 \qquad x_{i+1} = x_i^2 \qquad \boxed{(x_{i+1} \to 0) \text{ stable}}$

$x_i \geq 1 \qquad x_{i+1} = x_i^2 \qquad \boxed{(x_{i+1} \to \infty) \text{ unstable}}$

Math

Find fixed points $\quad x_0 = F(x_0)$

$$x_0 - F(x_0) = 0$$
$$x_0 - x_0^2 = 0$$
$$x_0(1 - x_0) = 0$$

→ Two solutions : $x_0 = 0$ and $x_0 = 1$
or 2 fixed points

Stability $\qquad F(x) = x^2 \qquad J = \dfrac{\partial F}{\partial x} = 2x$

$$\left.\begin{array}{lllll} J\big|_{x=0} = 2(0) = 0 & < 1 & x_0 = 0 & \text{stable} \\ J\big|_{x=1} = 2(1) = 2 & > 1 & x_0 = 1 & \text{unstable} \end{array}\right\}$$