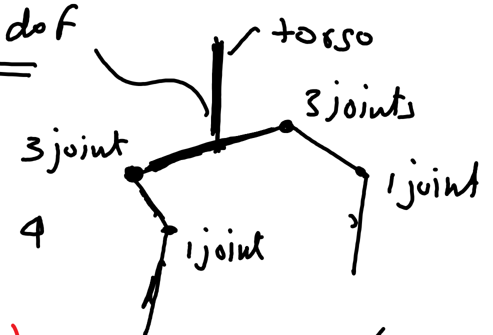
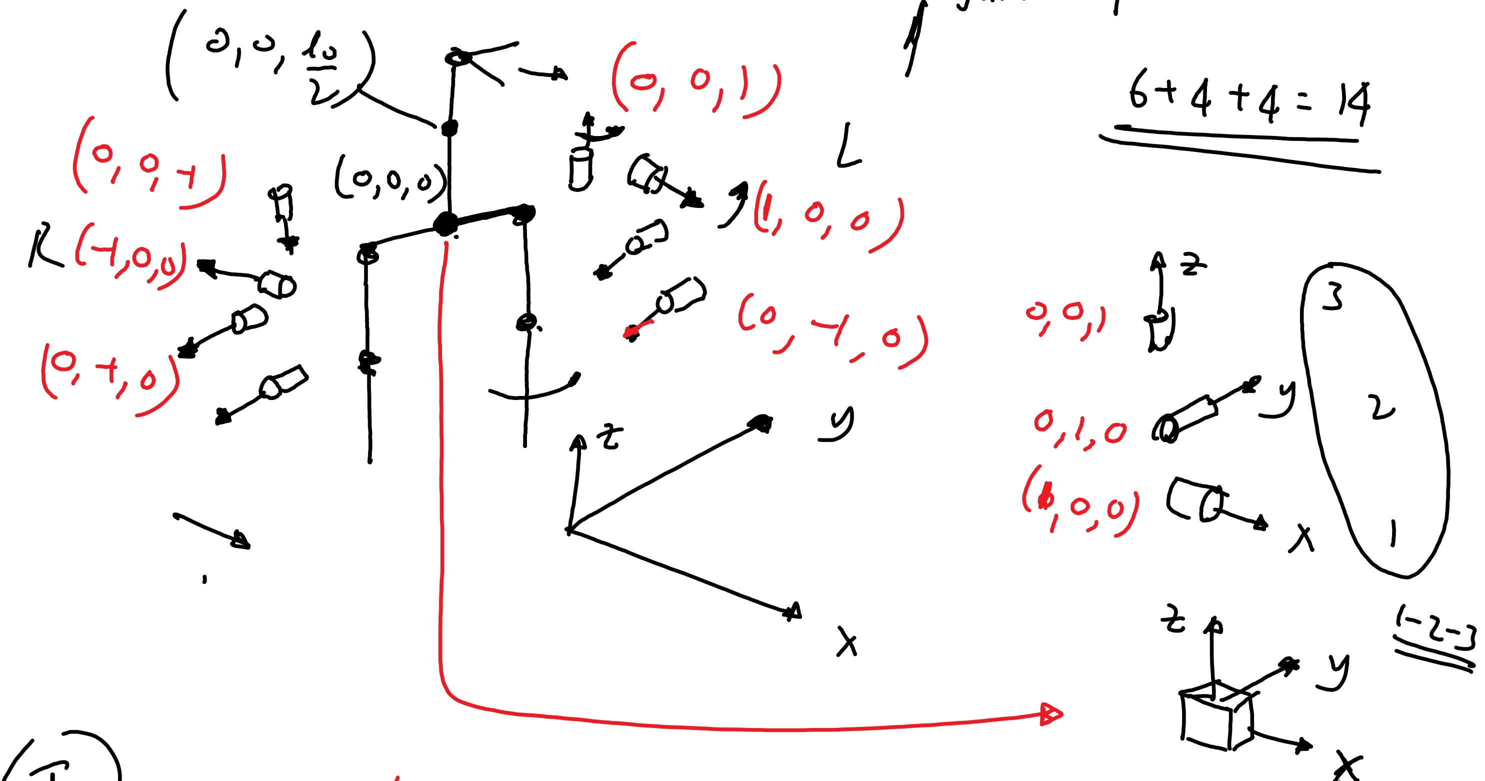
3 trans, 3 dof

6

3D - biped



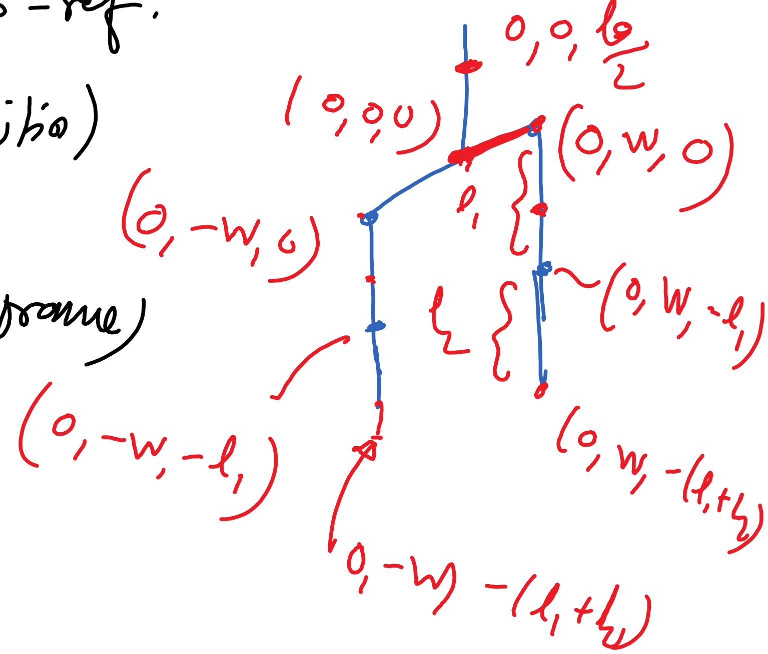


I

① position vectors: zero-ref.

② lin velocity : diff (position)
jacobian

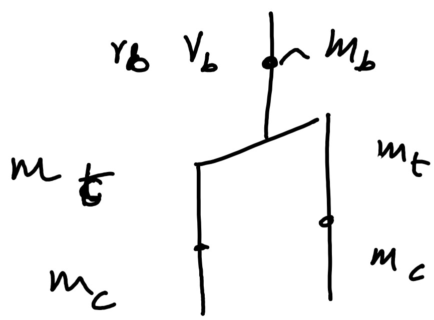
③ angular velocity (body frame)
recursive
formula

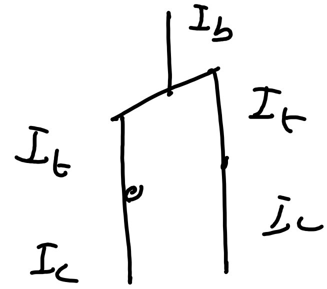


II Find Lagrangian

$$\mathcal{L} = T - V$$

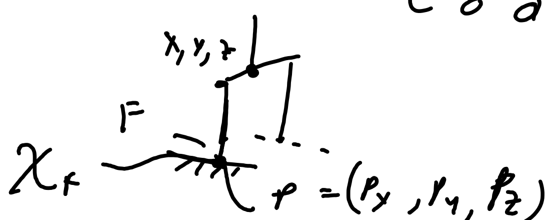
$$T = 0.5 \sum m(v \cdot v) + 0.5 \sum \Omega^T (I \Omega)$$





$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

$$V = \sum m g r$$



III EOM.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = Q_j + J^T p \quad \text{— (A)}$$

$$\chi_f = (x_f, y_f, z_f) = \text{constant}$$

$$\frac{d\chi_f}{dt} = \frac{\partial \chi_f}{\partial q}\dot{q} = 0 \Rightarrow J\dot{q} = 0$$

$$J\ddot{q} + \dot{J}\dot{q} = 0 \Rightarrow J\ddot{q} = -\dot{J}\dot{q} \quad \text{— (B)}$$

From (A)

$$M\ddot{q} + C\dot{q} + G = Q + \underset{14\times 3}{J^{T}}\, P_{3\times 1}$$

$$q = [\underline{x, y, z, \phi, \theta, \psi}, \underline{\phi_{lh}, \theta_{lh}, \psi_{lh}, \theta_{lk}}, \phi_{rh}, \theta_{rh}, \psi_{rh}, \theta_{rk}] \quad \text{— (C)}$$

$$Q = \text{external torque} = \underset{14\times 1}{\begin{bmatrix} zero(6,1) \\ T_1 \\ T_2 \\ \vdots \\ T_8 \end{bmatrix}}$$

$$= \underline{\underline{B}}\,\underbrace{T}_{8\times 1}$$

$$= \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & '' & & & & '' & \\ 0 & & & & & & '' & \\ 0 & & '' & & & & '' & \\ 0 & & & & & & '' & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ & & & & & & & \\ & & & & & & & 1 \end{array}\right] \begin{array}{l} \left.\vphantom{\begin{array}{c}0\\0\\0\\0\\0\end{array}}\right\} \text{6 rows} \\ \left.\vphantom{\begin{array}{c}0\\0\\0\\0\\0\\0\end{array}}\right\} \text{8 rows} \end{array}$$

From C

$$M\ddot{q} + C\dot{q} + G = BT + J^T P$$

From B

$$J\ddot{q} = -\dot{J}\dot{q}$$

$$\boxed{\underbrace{\begin{bmatrix} M & -J^T \\ J & 0 \end{bmatrix}}_{17\times17}\underbrace{\begin{bmatrix} \ddot{q}_{14\times1} \\ P_{3\times1} \end{bmatrix}}_{17\times1} = \underbrace{\begin{bmatrix} BT - C - G \\ -\dot{J}\dot{q} \end{bmatrix}}_{17\times1}}$$

single stance

code $\propto$) $M = A$

$-C-G = b$

Foot strike

$$v_{sw}^{+} = J_{sw}\dot{q}^{+} = 0$$

$$\int M\ddot{q} + \cancel{C} + \cancel{G} = \int B\cancel{T} + J_{st}^T P_{st} + J_{sw}^T P_{sw}\, dt$$

$$M(\dot{q}^{+} - \dot{q}^{-}) = J_{st}^T P_{st} + J_{sw}^T P_{sw}$$

$$J_{sw}\dot{q}^{+} = 0$$

✓ $$\boxed{\begin{bmatrix} M & -J_{sw}^T \\ J_{sw} & 0 \end{bmatrix}\begin{bmatrix} \dot{q}^{+} \\ P_{sw} \end{bmatrix} = \begin{bmatrix} J_{st}^T \underline{P_{st}} + M\dot{q}^{-} \\ 0 \end{bmatrix}}$$

Control partitioning | Partial feedback linearization

$$M\ddot{q} = -C - G + BT + J^T \underline{\underline{P}}$$
$$J\ddot{q} = -\dot{J}\dot{q}$$

$$\begin{bmatrix} M & +J^T \\ \underline{\underline{J}} & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \ddot{q} \\ P \end{bmatrix} = \begin{bmatrix} \underline{B}T - C - G \\ \underline{\underline{-\dot{J}}}\dot{q} \end{bmatrix}$$

17 X 1

$$\tilde{A}x = \tilde{b} + BT$$

can control only 8

$\underbrace{\theta, \phi, \psi, \theta_{lk}, \theta_{rk}}_{\text{fixed}}$ ~~to~~ $\underline{\underline{\theta_{rh}, \phi_{rh}, \psi_{rh}}}$ ↑ left stance

$\theta_{lh}, \phi_{lh}, \psi_{lh}$ ↑ right stance

$$q_c = 8 \text{ controlled dofs}$$

$$\rightarrow \quad q_c = S \begin{bmatrix} q \\ P \end{bmatrix}$$

selection matrix
(found by inspection)

$$\tilde{A}\chi = \hat{b} + BT \qquad \chi = \begin{bmatrix} \ddot{q} \\ p \end{bmatrix}$$

$$\ddot{q}_c = S \begin{pmatrix} \ddot{q} \\ p \end{pmatrix} = S\chi$$

$$\ddot{q}_C = S\chi = S\tilde{A}^{-1}(\hat{b} + BT) = v$$

?

$$= \ddot{q}_C^{ref} + K_p(q_C^{ref} - q_C) + K_d(\dot{q}_c^{ref} - \dot{q}_c)$$

$$\ddot{q}_c = V$$

$$S\tilde{A}^{-1}(\tilde{b} + BT) = V$$

$$S\tilde{A}^{-1}\tilde{b} + S\tilde{A}^{-1}BT = V$$

$$S\tilde{A}^{-1}BT = V - S\tilde{A}^{-1}b$$

$$\underline{\underline{T}} = (S\hat{A}^{-1}B)^{-1}(V - S\tilde{A}^{-1}B)$$