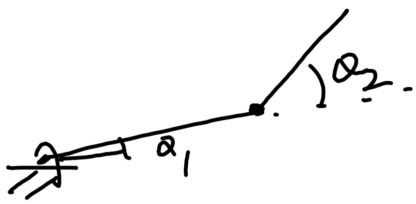
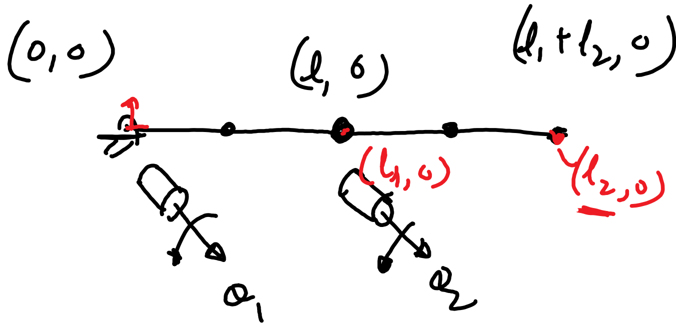
Zero reference model



Homogenous matrix



— world frame

Zero configuration

— homogenous transform

Prismatic joint

$$T = \begin{bmatrix} \overbrace{1 \quad 0 \quad 0}^{R} & sU_x \\ 0 \quad 1 \quad 0 & sU_y \\ 0 \quad 0 \quad 1 & sU_z \\ \underline{0 \quad 0 \quad 0} & \underline{1} \end{bmatrix}$$

default

$\Rightarrow$ $U = [U_x \quad U_y \quad U_z]$ are pix axis

$U_x^2 + U_y^2 + U_z^2 = 1$

e.g. linear motion along x-axis $U_x = 1$ $U_y = U_z = 0$

$\Rightarrow$ s — amount of translation

Revolute joint

$$R = \begin{bmatrix} u_x^2 V\phi + c\phi & u_x u_y V\phi - u_z s\phi & u_x u_z V\phi + u_y s\phi \\ u_x u_y V\phi + u_z s\phi & u_y^2 V\phi + c\phi & u_y u_z V\phi - u_x s\phi \\ u_x u_z V\phi - u_y s\phi & u_y u_z V\phi + u_x s\phi & u_z^2 V\phi + c\phi \end{bmatrix}$$

$$T = \begin{bmatrix} R_{3\times3} & (I - R)\,r \\ \underbrace{0 \quad 0 \quad 0}_{1\times3} & \underbrace{1}_{1\times1} \end{bmatrix} \qquad H = \begin{bmatrix} R & r \\ 0 & 1 \end{bmatrix}$$

local frame

I = 3X3 identity matrix

r = location of the joint location in the world frame

$c\phi = \cos\phi$, $s\phi = \sin\phi$, $V\phi = 1 - \cos\phi$

ϕ is the angle of rotation about the pin

$u = (u_x \quad u_y \quad u_z)$

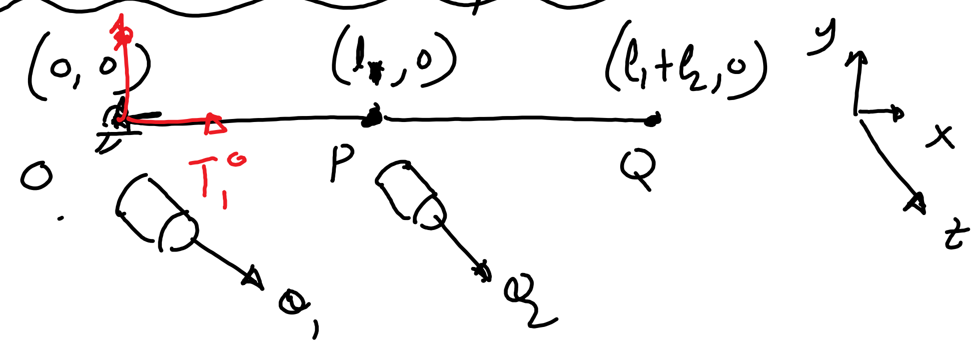
e.g. rotation about z-axis $\begin{array}{ccc} u_x & u_y & u_z \\ \end{array}$ $[0 \quad 0 \quad 1]$

Advantages of zero-ref world

① u_x, u_y, u_z need not be along any axis

② r is in global frame

① Double-link manipulator



(for O) $T_1^0 = \begin{bmatrix} R & (I-R)r \\ 0_{1\times3} & 1 \end{bmatrix}$

$\left.\begin{array}{l} v_x = v_y = 0 \quad v_z = 1 \\ \phi = \theta_1 \end{array}\right\} R$

$r = (0 \quad 0 \quad 0)^T$

(for P) $T_2^1 = \begin{bmatrix} R & (I-R)r \\ 0 & 1 \end{bmatrix}$

$v_x = v_y = 0 \quad v_z = 1$

$\phi = \theta_2$

$r = (l_1, 0, 0)^T$

$T_2^0 = T_1^0 \, T_2^1$

$=$

$\underline{\underline{P}}^0 = \underline{\underline{T}}_1^0 \, P^{\mathbf{0}}$ ← zero ref

$P^{\mathbf{0}} = [l_1 \quad 0 \quad 0]^T$

$P^0(\theta_1) \neq P'(\theta)$ — aside

$Q^0 = T_2^0 \, Q^{\mathbf{0}}$ ← world frame

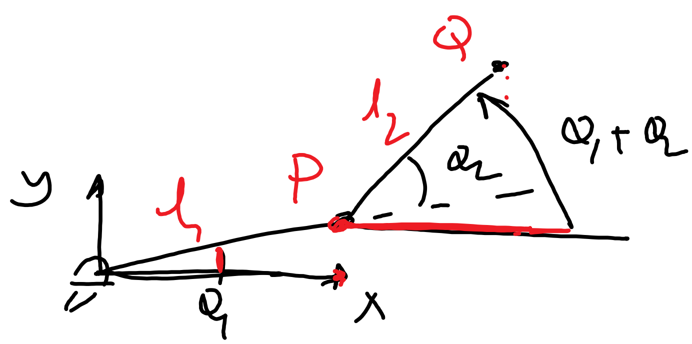
$Q^{\mathbf{0}}$ — zero-ref.

$[l_1 + l_2, 0, 0)^T$

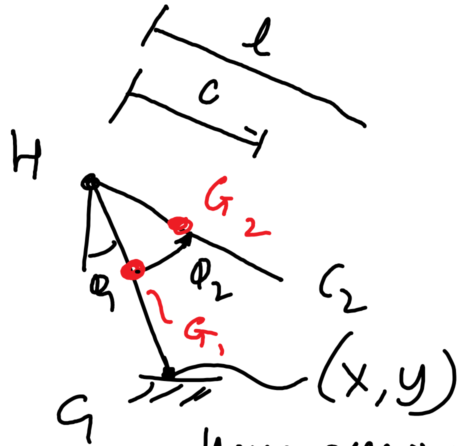
solving for P^0, Q^0 gives

$$P^0 = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ 0 \\ 1 \end{bmatrix}$$

$$Q^0 = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ 0 \\ 1 \end{bmatrix}$$

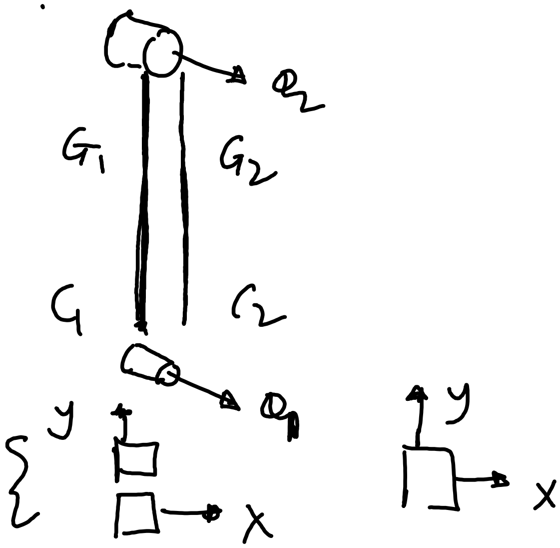


② Passive walker

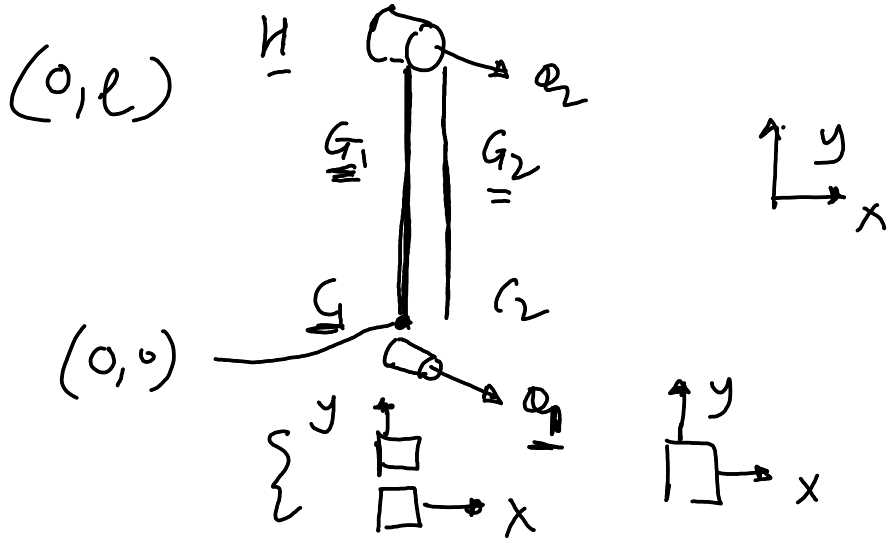


homogeneous transform.

Zero-ref.



$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$T_2^1 = \begin{bmatrix} R & (I-R)r \\ 0 & 1 \end{bmatrix} \quad \begin{aligned} &v_x = v_y = 0 \quad v_z = 1 \\ &\phi = \theta_1 \\ &r = [0 \ 0 \ 0]^T \end{aligned}$$

$$T_3^2 = \begin{bmatrix} R & (I-R)r \\ 0 & 1 \end{bmatrix} \quad \begin{aligned} &v_x = v_y = 0 \quad v_z = 1 \\ &\phi = \theta_2 \\ &r = [0 \ l \ 0]^T \end{aligned}$$

$$T_2^0 = T_1^0 \, T_2^1 \ ; \ T_3^0 = T_2^0 \, T_3^2$$

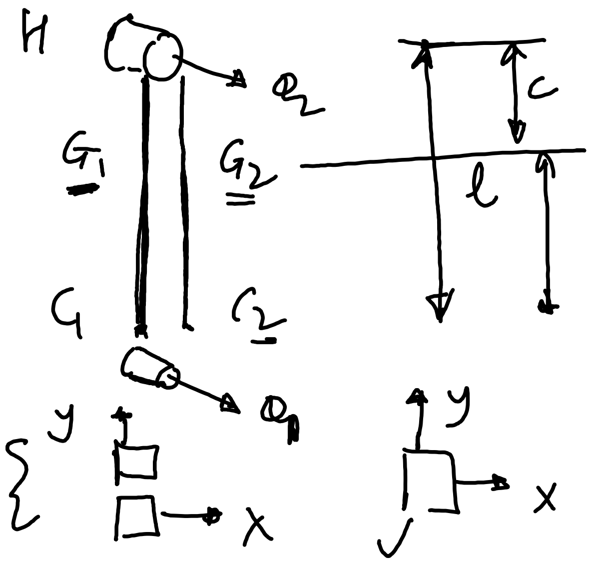
$$(C_1)^0 = (T_1^0)(C_1)_0$$

0 ← world

0 ← zero ref

$$= T_1^0 \, [0 \ 0 \ 0 \ 1]^T$$

$$= \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$$



$$(G_1)^0 = T_2^0 \, (G_1)_0^0$$

$$(G_1)^0 = T_2^0 \, [0 \; (l-c) \; 0 \; 1]^T$$

$$H^0 = T_2^0 \, (H)_0^0$$

$$= T_2^0 \, [0 \; l \; 0 \; 1]^T$$

$$(G_2)^0 = T_3^0 \, (G_2)_0^0$$

$$= (T_3^0) \, (0, (l-c), 0, 1)^T$$

$$(C_2)^0 = (T_3^0) \, (C_2)_0^0$$

$$= (T_3^0) \, [0, 0, 0, 1]^T$$

D-H — 4 param

Zero-ref

u_x, u_y

ϕ, r_x, r_y

5 parameter