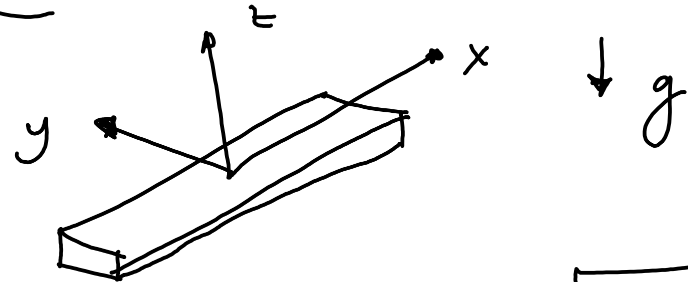
3D dynamics



Given initial speed & position ($v_x, v_y, v_z, \dot{\phi}, \dot{\theta}, \dot{\psi}, x, y, z, \phi, \theta, \psi$)
describe the motion of the object.

- Deriving the equations
- Simulate
- animate

Deriving equations

Euler-lagrange method.

① Position of center of mass. x, y, z,
3-2-1 euler : $\psi - \theta - \phi$

Velocity : $\dot{x}, \dot{y}, \dot{z}$

Angular velocity:

$$\Omega = \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta \sin\phi \\ 0 & -\sin\phi & \cos\theta \cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

② $\mathcal{L} = T - V$

$V = mgz$

$v = [\dot{x} \; \dot{y} \; \dot{z}]$, $mv^T v$

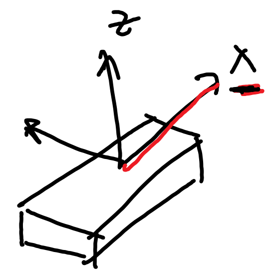
$$T = 0.5\, m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5\, \omega^T I_{fixed}\, \omega$$

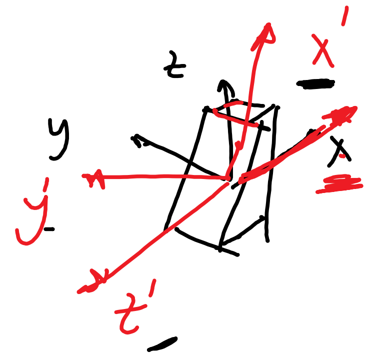
(complex)

$$+ 0.5\, \Omega^T I_{body}\, \Omega$$

I_{body}: does not change with orientation change; usually available in tables or CAD files

ω, Ω





fixed frame — body frame

$$T = 0.5\, m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5\, \Omega^T (I_{body}\, \Omega)$$

$$\Omega = \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} = f(\dot{\theta}, \dot{\phi}, \dot{\psi}, \theta, \phi, \psi)$$

body frame

③ $$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$$

$q_j = \{x, y, z, \phi, \theta, \psi\}$ – 6 dof

6 equation.

④ $$\underset{6\times 6}{A}\ \underset{6\times 1}{\ddot{q}} = \underset{6\times 1}{b}$$

$$\ddot{q} = A \backslash b$$