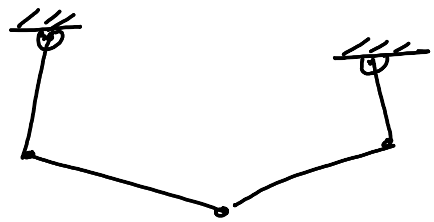
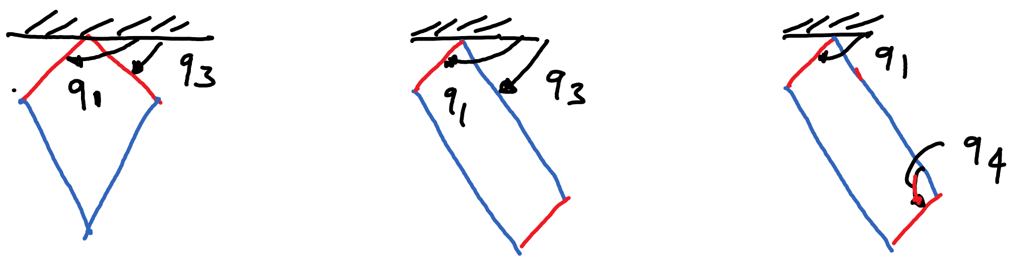
Closed-chain modelling, simulation, control.



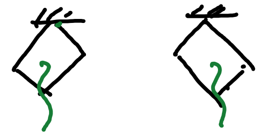
leg geometries

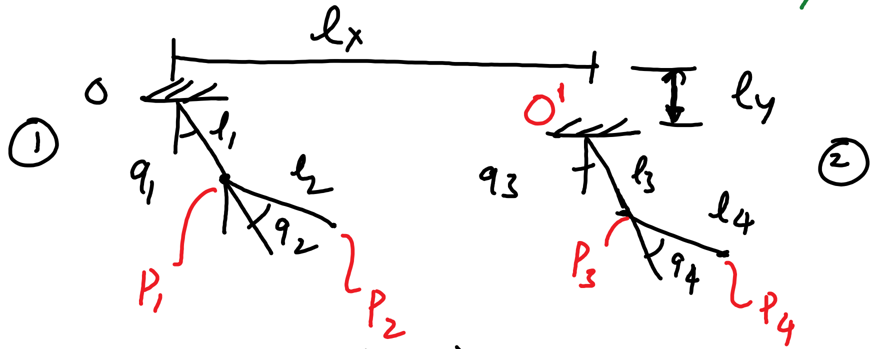


motor at q_1 & q_3 — motors at q_1 & q_3 — motors at q_1 & q_4

Minitaur — Atrias — Digit (?)

Modelling





$\rightarrow \mathcal{L}_1 = f_1(q_1, q_2, \dot{q}_1, \dot{q}_2)$

$\rightarrow \mathcal{L}_2 = f_2(q_3, q_4, \dot{q}_3, \dot{q}_4)$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}_i}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}_i}{\partial q_j} = \underset{=0}{Q_j} \Rightarrow$$

$q_j = q_1, q_2, q_3, q_4$.

$$\Rightarrow \boxed{\underset{4\times4}{M}\ \underset{4\times1}{\ddot{q}} + \underset{4\times1}{N} = 0}$$

I

$q = \{q_1\ q_2\ q_3\ q_4\}$

$x_{p_2} = $ ____ $\qquad x_{p_4} = $ ____

$y_{p_2} = $ ____ $\qquad y_{p_4} = $ ____

$X_{p_2} = \begin{Bmatrix} x_{p_2} \\ y_{p_2} \end{Bmatrix} \qquad X_{p_4} = \begin{Bmatrix} x_{p_4} \\ y_{p_4} \end{Bmatrix}$

For a chain: $X_{p_2} = X_{p_4}$

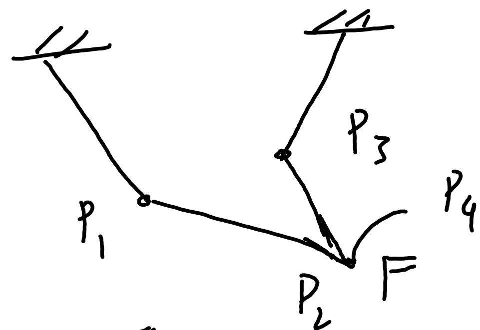
$$J_{p_2}\,\dot{q} = J_{p_4}\,\dot{q}$$

$$J_{p_2}\ddot{q} + \dot{J}_{p_2}\dot{q} = J_{p_4}\ddot{q} + \dot{J}_{p_4}\dot{q}$$

$$(J_{p_2} - J_{p_4})\ddot{q} = -\dot{J}_{p_2}\dot{q} + \dot{J}_{p_4}\dot{q} \quad \text{— II}$$

$$M\ddot{q} + N = 0 \qquad N = C(q,\dot{q})\dot{q} + G(q)$$

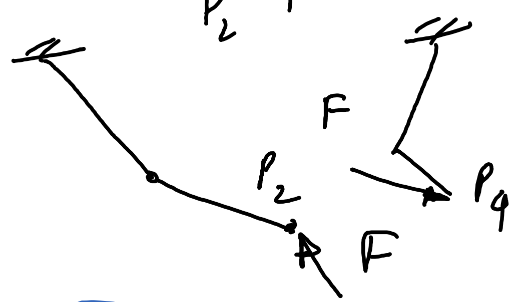
unconstrained



$$M\ddot{q} + N = J_{p_2}^T F + J_{p_4}^T(-F)$$

$$M\ddot{q} + N = (J_{p_2}^T - J_{p_4}^T)F$$

constrained system — III



from II & III

$$\begin{bmatrix} M & -(J_{p_2}^T - J_{p_4}^T) \\ (J_{p_2} - J_{p_4}) & 0 \end{bmatrix}\begin{bmatrix} \ddot{q} \\ F \end{bmatrix} = \begin{bmatrix} -N \\ (-\dot{J}_{p_2} + \dot{J}_{p_4})\dot{q} \end{bmatrix}$$

(*) Final equation.

$T_1 = T_2 = T_3 = T_4$

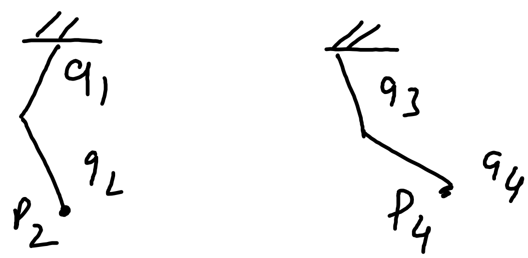
$$\begin{bmatrix} M & -(J_{p_2}^T - J_{p_4}^T) \\ (J_{p_2} - J_{p_4}) & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F \end{bmatrix} = \begin{bmatrix} -N \\ (-\dot{J}_{p_2} + \dot{J}_{p_4})\dot{q} \end{bmatrix}$$

$$\Big\downarrow \text{ Rewrite}$$

$$\begin{bmatrix} M & -J^T \\ J & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F \end{bmatrix} = \begin{bmatrix} -N \\ -\dot{J}\dot{q} \end{bmatrix}$$

Four link chain $\quad q = \{q_1 \quad q_2 \quad q_3 \quad q_4\}$ – 4 dofs

$F = \{F_x \quad F_y\}$



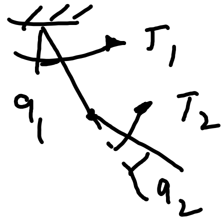
4 dofs

2 constraints.

net dof = $4 - 2 = 2$ ~ 2 torques.
that can
be applied

Recollect : feed back linearization

Open chain



$$M\ddot{q} + N = u \qquad u = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$$

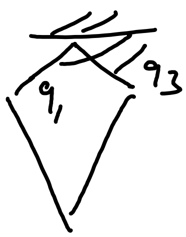
$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$u = M(\ddot{q}_{ref} + k_p(q_{ref} - q) + k_d(\dot{q}_{ref} - \dot{q})) + N$$

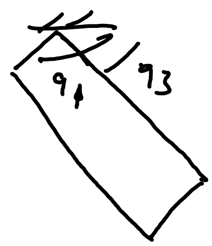
$$M(\ddot{q} - \ddot{q}_{ref} + k_d(\dot{q} - \dot{q}_{ref}) + k_p(q - q_{ref})) = 0$$

$$\ddot{e} + k_d\dot{e} + k_p e = 0 \qquad k_d = 2\sqrt{k_p}$$

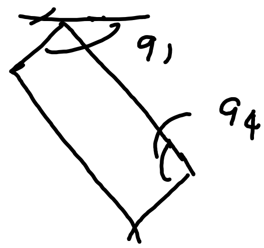
Closed chain



Minitaur



Atrias



Digit

$$\underset{6\times 6}{\begin{bmatrix} M & -J^T \\ J & 0 \end{bmatrix}} \underset{6\times 1}{\begin{bmatrix} \ddot{q} \\ F \end{bmatrix}} = \underset{6\times 1}{\begin{bmatrix} -N \\ -\dot{J}\dot{q} \end{bmatrix}}$$

① Figure out the controlled dofs (θ)

$$\theta = \begin{bmatrix} q_1 \\ q_3 \end{bmatrix} \quad \text{Minitaur/Atrias} \qquad \theta = \begin{pmatrix} q_1 \\ q_4 \end{pmatrix}$$

② Rewrite the equation

$$\begin{bmatrix} M & -J^T \\ J & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F \end{bmatrix} = \begin{bmatrix} -N \\ \dot{J}\dot{q} \end{bmatrix}$$

$$A_{11}\ddot{\theta} + A_{12} z = b_1$$

$$A_{21}\ddot{\theta} + A_{22} z = b_2$$

minitaur/Atrias $\quad \theta = \begin{bmatrix} q_1 \\ q_3 \end{bmatrix} \quad z = \begin{bmatrix} q_2 \\ q_4 \\ f_x \\ f_y \end{bmatrix}$

Digit $\quad \theta = \begin{bmatrix} q_1 \\ q_4 \end{bmatrix} \quad z = \begin{bmatrix} q_2 \\ q_3 \\ f_x \\ f_y \end{bmatrix}$

Solve for z

$$z = A_{22}^{-1}(b_2 - A_{21}\ddot{\theta})$$

sub.

$$A_{11}\ddot{\theta} + A_{12}A_{22}^{-1}(b_2 - A_{21}\ddot{\theta}) = b_1$$

$$\underbrace{(A_{11} - A_{12}A_{22}^{-1}A_{21})}_{\tilde{A}} \underbrace{\ddot{\theta}}_{\ddot{\theta}} = \underbrace{(b_1 - A_{12}A_{22}^{-1}b_2)}_{\tilde{b}}$$

$$\underline{\tilde{A}\ddot{\theta} = \tilde{b} + u}$$

control partitioning

$u = \begin{bmatrix} T_1 \\ T_4 \end{bmatrix}$ Digit

$u = \begin{bmatrix} T_1 \\ T_3 \end{bmatrix}$ minitaur/Atrias

$$u = \tilde{A}\left(\ddot{\theta}_{ref} + k_p(\theta_{ref} - \theta) + k_d(\dot{\theta}_{ref} - \dot{\theta})\right) - \tilde{b}$$