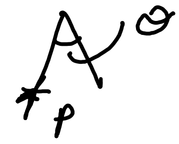
So far legged machines

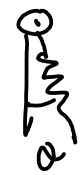
① Passive walkers

② Powered machines — walker



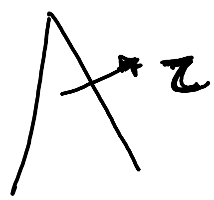
control P, θ ↑ foot placement

— hopper



θ = foot placement

P, θ → controlled once per step.



← set the torque

Low-level or joint level control, find $\tau(t)$

↓

Trajectory tracking

↓

① PID

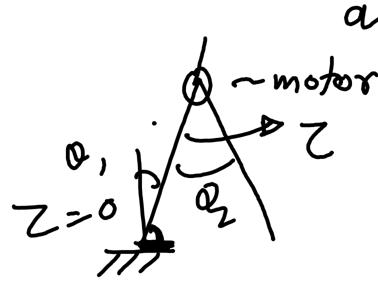
② Control partitioning

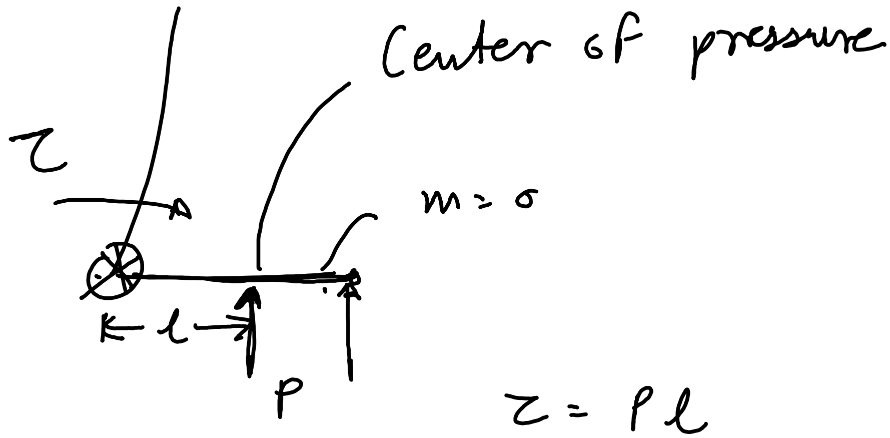
| issue

under-actuation

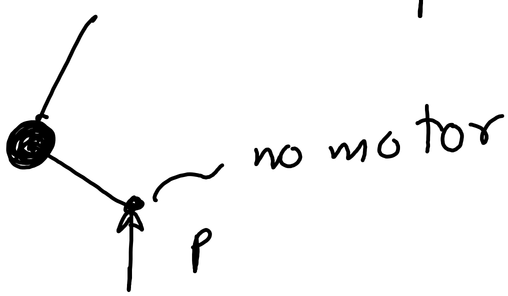
actuator < joint

τ θ_1, θ_2





$$\tau = p\ell$$



Walker with a hip torque

$$u = \tau$$

$$\underset{\cancel{2\times1}}{M(q)}\,\underset{2\times1}{\ddot q} + \overbrace{\underset{2\times1}{C(q,\dot q)\dot q} + \underset{2\times1}{G(q)}}^{N} = \begin{bmatrix}0\\ \tau\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$= Bu$$

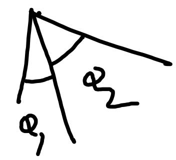
$$M\ddot q + N(q,\dot q) = Bu$$

selection matrix

Use partial feedback Linearization

θ_1 , θ_2

× ✓



Try to track θ_2 as best as we can (joint-level)

⇒ Then hope that θ_1 is stable over the time scale of a step.

$$\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2 = 4$$

Poincare map $4-1=3$ eigenvalues $=3$

Using PFL : 1 = eigenvalue

$$M\ddot{\theta} + N = Bu$$

$$\ddot{\theta} = M^{-1}(Bu - N) \qquad -(1)$$

$$\theta = (\theta_1, \theta_2) = [\theta_u, \theta_c]$$

$$\theta_c = \theta_2 = [0 \ 1] \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = S_c\,\theta$$

$$\Rightarrow \quad \ddot{\theta}_c = S_c\ddot{\theta} = S_c M^{-1}(Bu - N) \quad \text{From } (1)$$

$$= \underbrace{\ddot{\theta}_c^{ref} + k_p(\theta_c - \theta_c^{ref}) - k_d(\dot{\theta}_c - \dot{\theta}_c^{ref})}_{=V} \quad (2)$$

$$S_c M^{-1}(Bu - N) = V$$

$$S_c M^{-1}Bu - S_c M^{-1}N = V$$

$$S_c M^{-1}Bu = V + S_c M^{-1}N$$

$$\boxed{u = (S_c M^{-1}B)^{-1}(V + S_c M^{-1}N)}$$

This is the control that ensure $\theta_c = \theta_c^{ref}$