1) Set-point control

$\rightarrow \quad M\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau$

$\rightarrow$ set-point $\quad \theta = \theta_d$

2 options

$\rightarrow$ ① $\tau = -k_p(\theta - \theta_d) - k_d\dot{\theta}$ (Coppelia sim)

$\rightarrow$ ② $\tau = M(-k_p(\theta-\theta_d) - k_d\dot{\theta}) + C(\theta,\dot{\theta}) + G(\theta)$

n decoupled system $\quad k_d = 2\sqrt{k_p}$

examples

(i) random / fictitious 2x2

(ii) single link pendulum

a) P-D ①

b) control partitioning ②

Trajectory tracking

$\theta_{ref}, \dot{\theta}_{ref}, \ddot{\theta}_{ref}$ (e.g. fifth order polynomial)

$$\tau = M\left(\ddot{\theta}_{ref} - k_p(\theta - \theta_{ref}) - k_d(\dot{\theta} - \dot{\theta}_{ref})\right) + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) \quad -①$$

$$M\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau \quad -②$$

Put ① in ②

$$\Rightarrow M\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + \cancel{G(\theta)} = M(\ddot{\theta}_{ref} - k_p(\theta - \theta_{ref}) - k_d(\dot{\theta} - \dot{\theta}_{ref})) + C(\theta, \dot{\theta}) + \cancel{G(\theta)}$$

$$\Rightarrow M\left[(\ddot{\theta} - \ddot{\theta}_{ref}) + k_p(\theta - \theta_{ref}) + k_d(\dot{\theta} - \dot{\theta}_{ref})\right] = 0$$

$$e = \theta - \theta_{ref}$$

$$\Rightarrow M[\underbrace{\ddot{e} + k_d\dot{e} + k_p e}_{\text{spring-mass-damper}}] = 0$$

$$k_d = 2\sqrt{k_p}$$

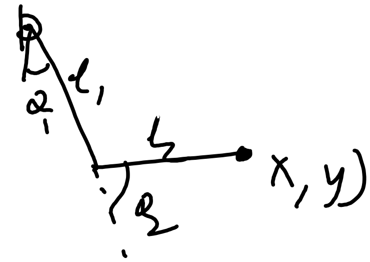
Alternate

$$\tau = -k_p(\theta - \theta_{ref}) - k_d(\dot{\theta} - \dot{\theta}_{ref})$$

Naive P-D controller

(3) Cartesian-based tracking

$q = \{x, y\}$



$q_{ref} = \{x_{ref}, y_{ref}\}$

$q_{ref}, \dot{q}_{ref}, \ddot{q}_{ref}$ ⟵ given

Forward kinematics : $q_{ref} = f(\theta_{ref})$ — I

Inverse kinematics : $\theta_{ref} = f^{-1}(q_{ref})$

1 use fsolve

Differentiate eq^{n} I

$$\dot{q}_{ref} = \frac{df}{dt}(\theta_{ref})$$

$$\dot{q}_{ref} = J\,\dot{\theta}_{ref} \qquad \left(J = \frac{\partial f}{\partial \theta}\right) \quad \text{II}$$

2 — $\dot{\theta}_{ref} = (J^{-1})\,\dot{q}_{ref}$

Differentiate II

$$\ddot{q}_{ref} = J\,\ddot{\theta}_{ref} + \dot{J}\,\dot{\theta}_{ref}$$

3 — $\ddot{\theta}_{ref} = J^{-1}\{\ddot{q}_{ref} - \dot{J}\,\dot{\theta}_{ref}\}$

$$\overbrace{q_{ref}, \dot{q}_{ref}, \ddot{q}_{ref}}^{\text{given}} \xrightarrow[3]{\text{Use } 1, 2} \theta_{ref}, \dot{\theta}_{ref}, \ddot{\theta}_{ref}$$

Stage 1

$$\underbrace{q_{ref},\ \dot{q}_{ref},\ \ddot{q}_{ref}}_{\text{given}} \xrightarrow[\text{③}]{\text{Use ①, ②}} \theta_{ref},\ \dot{\theta}_{ref},\ \ddot{\theta}_{ref}$$

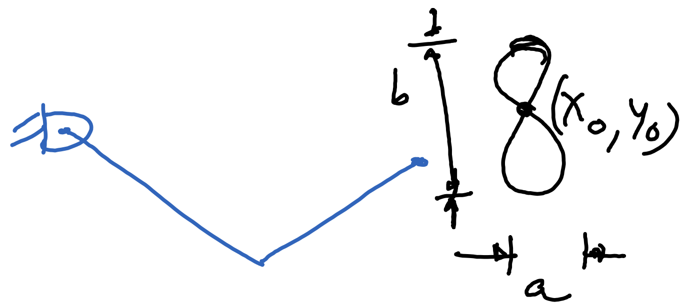
Stage 2

$$\tau = M\left[\ddot{\theta}_{ref} - K_p(\theta - \theta_{ref}) - K_d(\dot{\theta} - \dot{\theta}_{ref})\right] + C(\theta, \dot{\theta})\dot{\theta} + G(\theta)$$

Example

$\rightarrow x = x_0 + A \sin(a\tau)$

$\rightarrow y = y_0 + B \cos(b\tau)$



Leminscate

$q = \{x, y\}$

$q_{ref} = ?$ $\qquad \tau = 0, 2\pi$

(x_{ref}, y_{ref})

$\dot{x} = A a \cos(a\tau) \dfrac{d\tau}{dt}$ $\quad$ ($\dfrac{d\tau}{dt} = \dot{\tau}$) $\qquad \dot{x} = 0 \quad \dot{\tau} = 0$

$\dot{y} = -B b \sin(b\tau) \dfrac{d\tau}{dt}$ $\qquad \dot{y} = 0 \quad \dot{\tau} = 0$

$\ddot{x} = $ _ __ __ $\qquad \ddot{x} = 0 \quad \ddot{\tau} = 0$

$\ddot{y} = $ _ _ $\qquad \ddot{y} = 0 \quad \ddot{\tau} = 0$

$\tau = 0 \ , \ 2\pi$ $\quad$ x_{ref}, y_{ref} start, end position

$\dot{\tau} = 0$ at $\tau = 0, 2\pi$ $\quad$ $\dot{x}_{ref}, \dot{y}_{ref} = 0$ $\quad$ "

$\ddot{\tau} = 0$ at $\tau = 0, 2\pi$ $\quad$ $\ddot{x}_{ref}, \ddot{y}_{ref} = 0$ $\quad$ "

$\tau = \underline{a_0} + \underline{a_1} t + \underline{a_2} t^2 + \underline{a_3} t^3 + \underline{a_4} t^4 + \underline{a_5} t^5$

6 conditions & 6 constants

Find a's

$x_{ref}, y_{ref}, \dot{x}_{ref}, \dot{y}_{ref}, \ddot{x}_{ref}, \ddot{y}_{ref}$ ✓

Control schemes

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau$$

① **P-I-D control**

$$\tau = -k_p(\theta-\theta_{ref}) - k_d(\dot{\theta}-\dot{\theta}_{ref}) - k_i\int(\theta-\theta_{ref})dt$$

② **Control partitioning**

$$\tau = M(\theta)\left[\ddot{\theta}_{ref} - k_p(\theta-\theta_{ref}) - k_d(\dot{\theta}-\dot{\theta}_{ref})\right] + \underline{C(\theta,\dot{\theta})\dot{\theta} + G(\theta)}$$

sensor measurement

③ **Feed Forward**

$$\tau = \tau_{ff} - k_p(\theta-\theta_{ref}) - k_d(\dot{\theta}-\dot{\theta}_{ref})$$

$$\tau_{ff} = \underline{M(\theta_{ref})\ddot{\theta}_{ref} + C(\theta_{ref},\dot{\theta}_{ref})\dot{\theta}_{ref} + G(\theta_{ref})}$$

pre-computed

④ **P-D + gravity compensation**

$$\tau = -k_p(\theta-\theta_{ref}) - k_d(\dot{\theta}-\dot{\theta}_{ref}) + G(\theta)$$

good when manipulator is in vertical plane & moving slowly.

When manipulator moves fast $C(\theta,\dot{\theta})\dot{\theta}$ is substantially

substantially