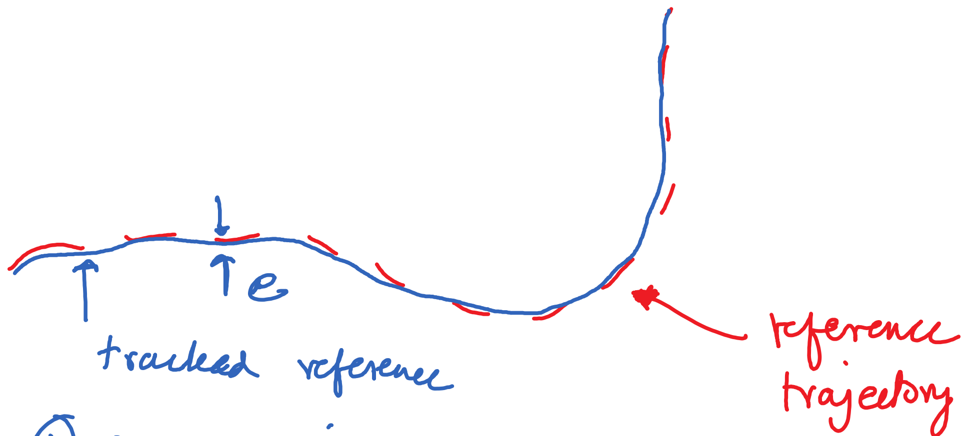
Feedback Control



1. sensor noise
2. disturbances.
3. modelling inaccuracies

Manipulators

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$$

Same form

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau$$

?

simplest, 1 dof

$$m\ddot{x} + c\dot{x} + kx = F$$

Uncontrolled spring-mass-damper

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad \{F = 0\}$$

$$\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{k}{m}\right)x = 0$$

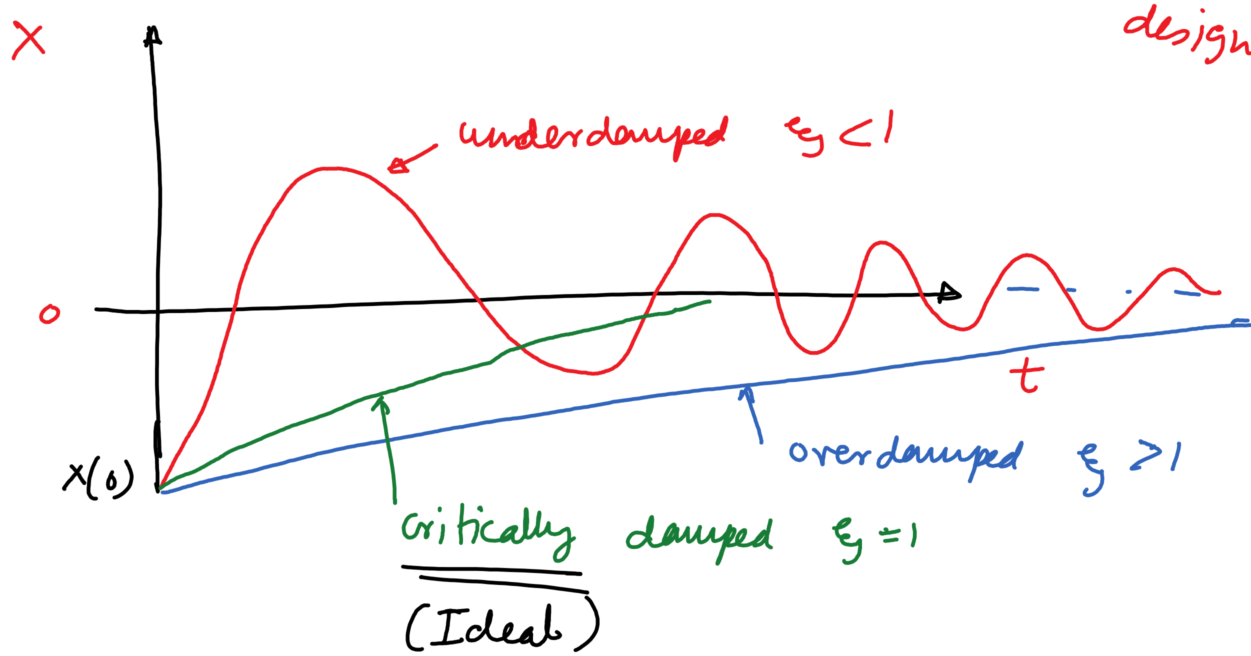
$\omega_n^2 = \frac{k}{m}$ $\qquad 2\xi\omega_n = \frac{c}{m}$

$c, m, k \longrightarrow$ 2 numbers $\frac{c}{m}, \frac{k}{m}$

2 ξ, ω_n

$\omega_n = \sqrt{k/m}$ $\qquad \xi = \frac{c}{2\sqrt{mk}} = 1 \Rightarrow \boxed{c = 2\sqrt{mk}}$

physically design



Controlled case

$$m\ddot{x} + c\dot{x} + kx = F$$

Choose $F = -k_p x - k_d \dot{x}$ $\quad$ k_p, k_d constants
feedback

$$m\ddot{x} + c\dot{x} + kx = -k_p x - k_d \dot{x}$$

$$m\ddot{x} + (c + k_d)\dot{x} + (k + k_p)x = 0$$

$$\zeta = \frac{(c + k_d)}{2\sqrt{m(k + k_p)}} = 1$$

find k_p, k_d

$$(c + k_d) = 2\sqrt{m(k + k_p)}$$

Square $(c + k_d)^2 = 4m(k + k_p)$

1 eq^n $\quad c^2 + 2k_d c + k_d^2 = 4mk + 4mk_p$

2 variables

$$k_d^2 + 2k_d c + (c^2 - 4mk - 4mk_p) = 0$$

$$k_d = \frac{-2c \pm \sqrt{(2c)^2 - 4(1)(c^2 - 4mk - 4mk_p)}}{2}$$

$$k_d = -c \pm 2\sqrt{m(k + k_p)}$$

$$k_d = -c \pm 2\sqrt{m(k+k_p)}$$

$$\boxed{k_d = -c + 2\sqrt{m(k+k_p)}}$$

Solution
critically damped

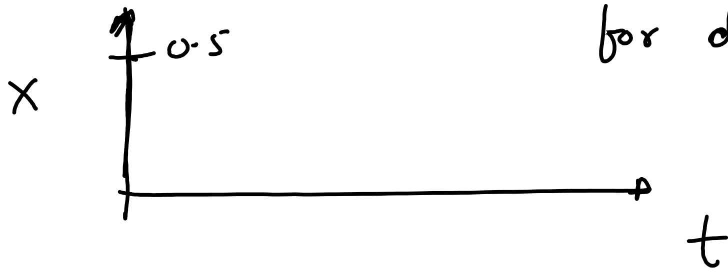
MATLAB example:

$$m=1,\ c=1,\ k=10,\ F=-k_d\dot{x}-k_p\dot{x}$$

$$\rightarrow\ k_d=-1+2\sqrt{10+k_p}\quad -\ 1$$

$$x(0)=0.5,\ \dot{x}(0)=0$$

for different k_p's, k_d from I



Extending to 2-dof

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = -\begin{bmatrix} k_{d11} & k_{d12} \\ k_{d21} & k_{d22} \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} - \begin{bmatrix} k_{p11} & k_{p12} \\ k_{p21} & k_{p22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

1 eq^n = 1 dof — 2 unknowns k_p, k_d

2 eq^n = 2 dof — 8 unknowns $[K_p]_{2\times2}$ $(K_d)_{2\times2}$

Control partitioning / computed torque control / feedback linearization

$$M(\underline{\theta})\ddot{\theta} + C(\underline{\theta, \dot{\theta}})\dot{\theta} + \underline{G}(\theta) = \tau$$

Assume $\tau = \hat{M}(\theta)[-k_p\theta - k_d\dot{\theta}] + \underline{\underline{\hat{C}(\theta,\dot{\theta})}}\dot{\theta} + \underline{\underline{\hat{G}(\theta)}}$

^ are estimates. $\Rightarrow$ perfect estimate $M \approx \hat{M}$

$$C \approx \hat{C}$$

$$G \approx \hat{G}$$

Assume perfect estimates

$$M\ddot{\theta} + \cancel{C}\dot{\theta} + \cancel{G} = M[-k_p\theta - k_d\dot{\theta}] + \cancel{C}\dot{\theta} + \cancel{G}$$

$$M(\ddot{\theta} + k_d\dot{\theta} + k_p\theta) = 0$$

$M \neq 0$

$$\ddot{\theta} + K_d \dot{\theta} + K_p \theta = 0$$

$$\Downarrow$$

decoupled, linear equation

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \vdots \\ \ddot{\theta}_n \end{bmatrix} + \begin{bmatrix} K_{d_1} & 0 & 0 & \cdots & \\ 0 & K_{d_2} & \cdots & & \\ 0 & 0 & K_{d_3} & \cdots & \\ & & & \ddots & \\ & & & & K_{d_n} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} + \begin{bmatrix} K_{p_1} & 0 & \cdots & \\ 0 & K_{p_2} & 0 & \\ & & \ddots & \\ & & & K_{p_n} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \vdots \\ \theta_n \end{bmatrix} = 0$$

$$\left.\begin{aligned} &\ddot{\theta}_1 + K_{d_1} \dot{\theta}_1 + K_{p_1} \theta_1 = 0 \\ &\ddot{\theta}_2 + K_{d_2} \dot{\theta}_2 + K_{p_2} \theta_2 = 0 \\ &\vdots \\ &\ddot{\theta}_n + K_{d_n} \dot{\theta}_n + K_{p_n} \theta_n = 0 \end{aligned}\right\} \text{Linear, decoupled.}$$

$m = 1, \; c = 0, \; k = 0$

$$\boxed{K_{d_i} = 2\sqrt{K_{p_i}}}$$