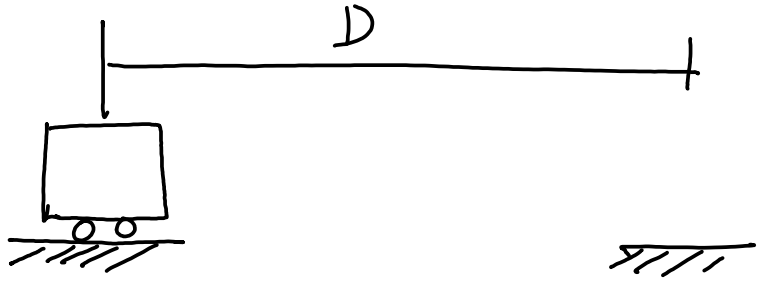
Trajectory optimization



$x_1 = 0$ (position) $\qquad x_1 = D$

$x_2 = 0$ (velocity) $\qquad x_2 = 0$

minimize time taken $\int_0^T dt = T$

trying to find $u(t) = ?$
control

Cost $\int_0^T dt = T$

Dynamics

$\ddot{x} = u$

$\downarrow$

$\dot{x}_1 = x_2$

$\dot{x}_2 = u$ } (given)

Constraints:

$x_1(0) = 0$

$x_2(0) = 0$

$x_1(T) = D$

$x_2(T) = 0$

$\rightarrow \quad -v_{max} \leq v \leq v_{max}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = u$

$u(t) = ?$

Formulate

Two ways:

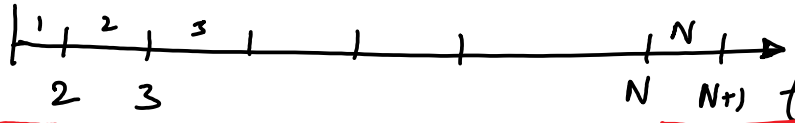
Trajectory optimization → Parameter optimization

Infinite dimension → Finite dimension

① Collocation or transcription method

② Shooting method

① Collocation or transcription method



① $x_1(1)$ $x_1(2)$ ...

$x_2(1)$, $x_2(2)$, ...

$u(1)$, $u(2)$, ...

$x_1(N+1)$

$x_2(N+1)$

$u(N+1)$

position, velocity } $3(N+1)$

optimization variables / T — optimization variable

② T constraints

cost

③ $x_1(1) = x_2(1) = x_2(N+1) = 0 \quad x_1(N+1) = p$

$-u_{max} \le u(i) \le u_{max}$

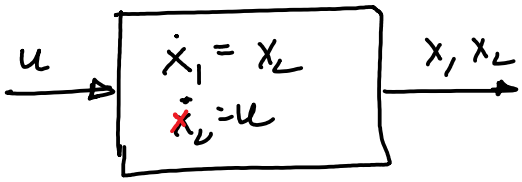
$\dot{x}_1 = x_2 \Rightarrow x_1(i+1) - x_1(i) = x_2(i)\,dt \quad i = 1, \dots N$

$\dot{x}_2 = u \Rightarrow x_2(i+1) - x_2(i) = u(i)\,dt \quad i = 1, \dots N$ (Euler's)

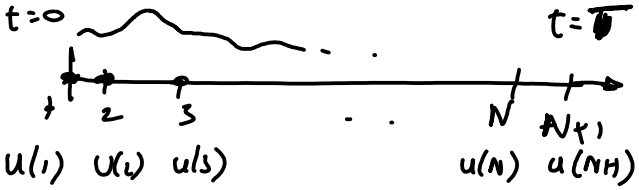
$x_2(i+1) - x_2(i) = 0.5\,(u(i) + u(i+1))\,dt$

$dt = \frac{T}{N+1}$

② Shooting method



blackbox (integration)



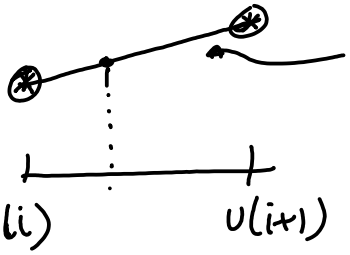
initial condition

⇒ $x_1(0) = x_2(0) = x_2(T) = 0$, $x_1(T) = D$

constraints ⇓ $x_2(N+1) = 0$ $x_1(N+1) = D$

⇒ T (cost minimized)

⇒ $T, u(1), u(2), \ldots u(N), u(N+1)$ optimization variable



interp1 (t_{data}, Y_{data}, t_{query})

$= Y_{query}$.