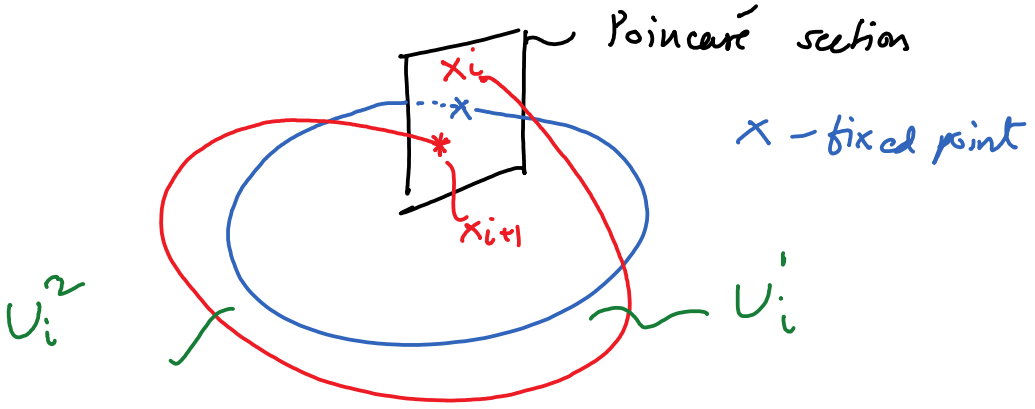
Controlling the hopper

Passive walker / hopper



$$x_{i+1} = F(x_i)$$

F - Poincaré map

$u_i^{*,2}$ e.g. foot placement angle
spring force

$$x_{i+1} = F(x_i, u_i)$$

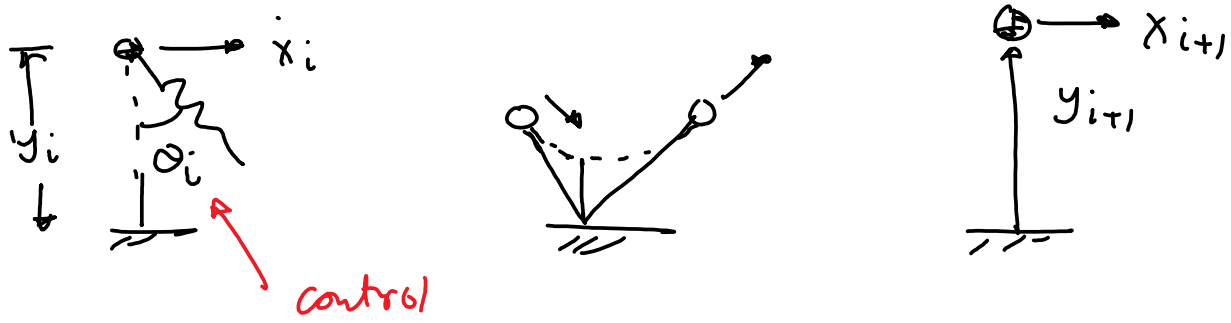
controlled Poincaré map

controls

Controls: ⟶ set once-per-step.

Discrete-control
very low bandwidth
~ 1 - 4 Hz
or times per second.

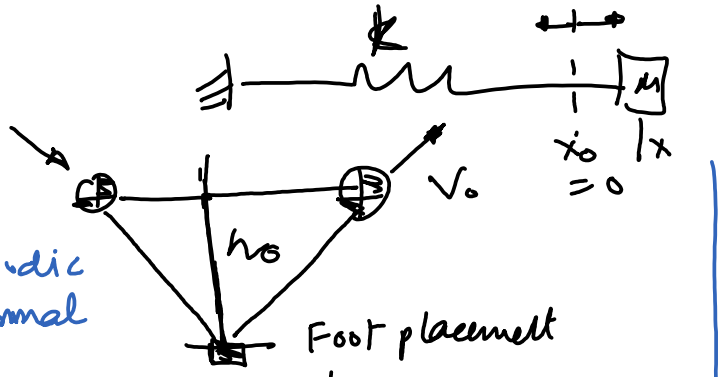
Control of hopper

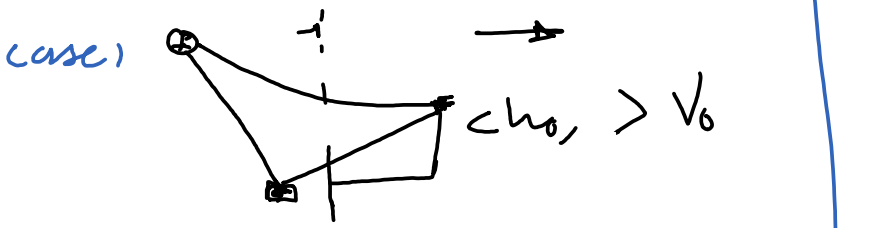


$$X_{i+1} = F(X_i, U_i)$$

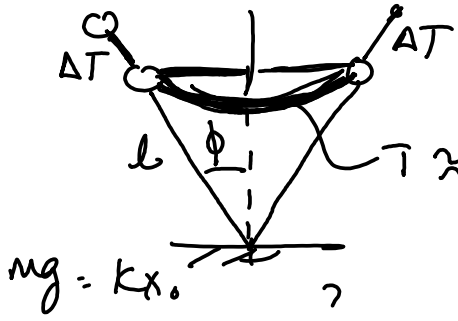
(x_{i+1}, y_{i+1}) $(\dot{x}_0, y_i)$ θ_i

Raibert controller
Boston Dynamics





$$T = 2\pi\sqrt{\frac{m}{k}}$$



$T + 2\Delta T$

$2l\sin\phi = \dot{x}_0(T + 2\Delta T)$

$\phi = \sin^{-1}\left(\frac{\dot{x}_0(T+2\Delta T)}{2l}\right)$

$T = \pi\sqrt{\frac{m}{k}}$

$$\phi = \sin^{-1}\left(\frac{\dot{x}_0 (T + 2\Delta T)}{2\ell}\right)$$

$$\phi \simeq \sin^{-1}\left(\frac{\dot{x}_0 T}{2\ell}\right) + \text{correction}$$

$$\phi \simeq \sin^{-1}\left(\frac{\dot{x}_0 T}{2\ell}\right) + K(\dot{x}_0 - \dot{x}_{des})$$

K: gain

$\dot{x}_{des}$: desired apex horizontal velocity

$$T = \pi\sqrt{\frac{M}{K}}$$

Foot placement angle

$$\theta = \phi = \sin^{-1}\left(\frac{\dot{x}_0 \pi}{2\ell}\sqrt{\frac{m}{K}}\right) + K(\dot{x}_0 - \dot{x}_{des})$$

To get the hopper have an apex horizontal velocity of $\dot{x}_{des}$.