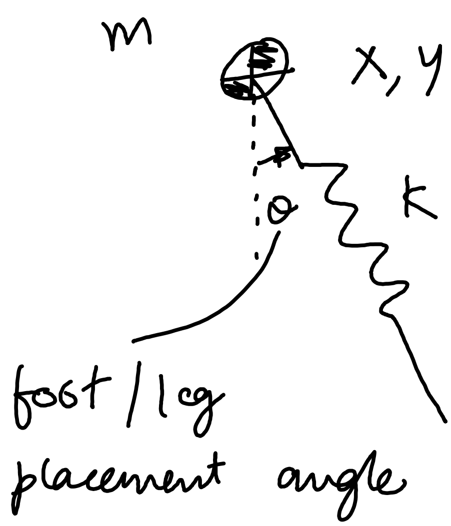
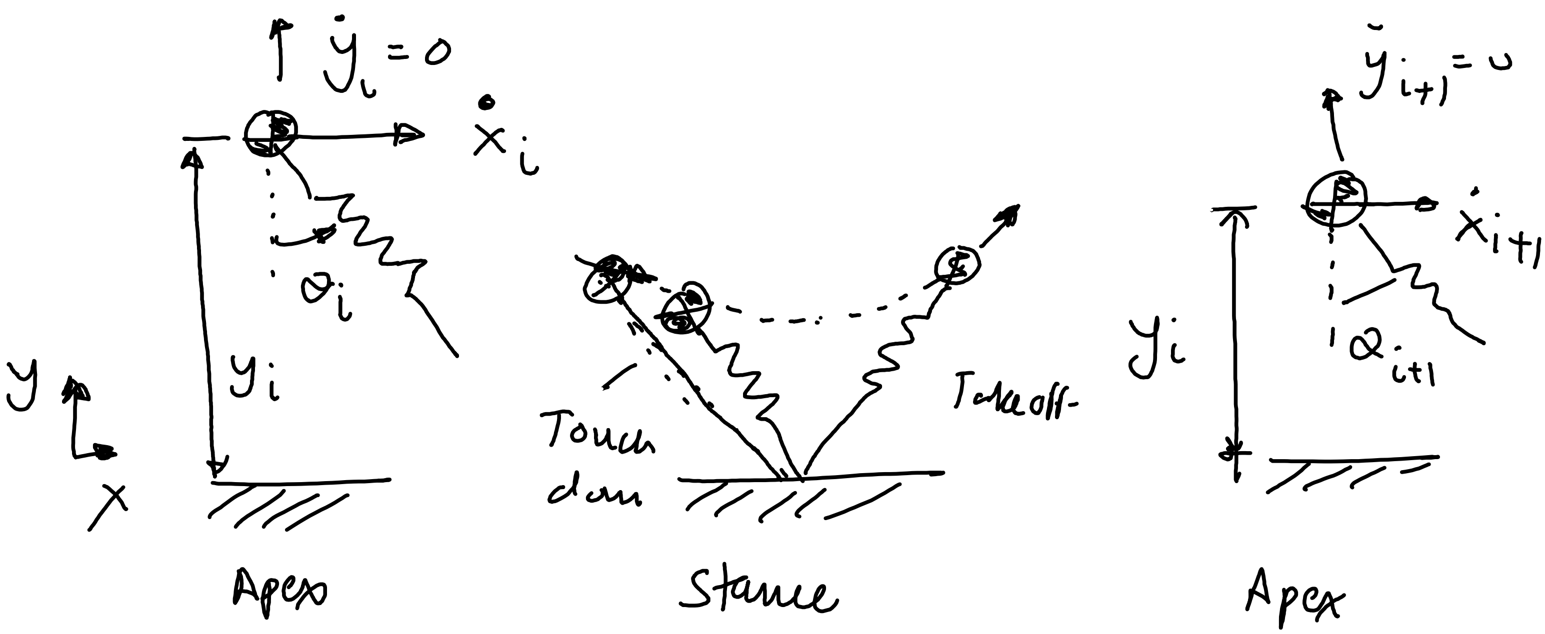
Hopping model – Spring-Loaded Inverted Pendulum (SLIP)





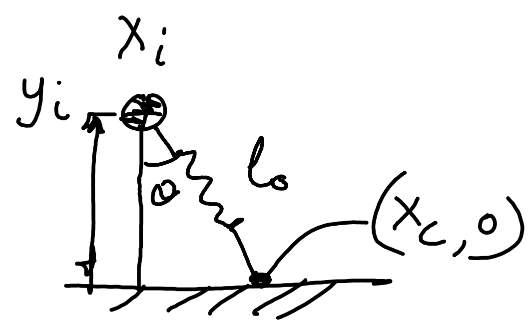
$\xrightarrow{\text{Apex}}$ Apex–stance $\xrightarrow{\text{Touchdown}}$ Stance $\xrightarrow{\text{Take-off}}$ Stance–Apex $\xrightarrow{\text{Apex}}$

One-step.

$$\underset{i}{\xrightarrow{\text{Apex}}} \text{ Apex-stance } \xrightarrow{\text{Touchdown}} \text{ Stance } \xrightarrow{\text{Take-off}} \text{ Stance-Apex } \underset{i+1}{\xrightarrow{\text{Apex}}}$$

One-step.

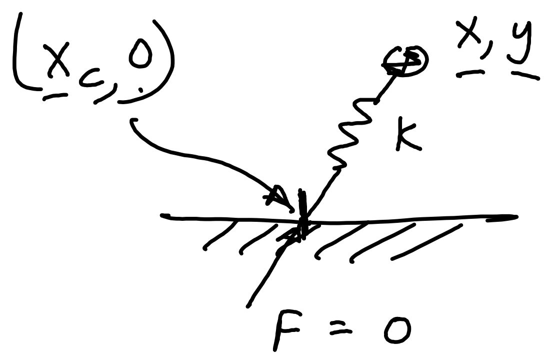
① Apex : $\boxed{\dot{y}_i = \dot{y}_{i+1} = 0}$ ✓



② Touch down

$\boxed{y_i - l_0 \cos\theta = 0}$ ✓

$\Longrightarrow \quad x_c = x_i + l_0 \sin\theta$



③ Take-off

$F = k(l - l_0) = 0$

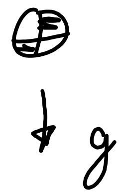
$\Rightarrow l = l_0$

$l^2 = (x - x_c)^2 + (y - 0)^2 \Rightarrow l = \sqrt{(x - x_c)^2 + y^2}$

$$\boxed{\sqrt{(x - x_c)^2 + y^2} - l_0 = 0}$$

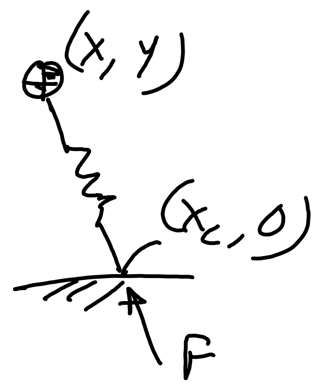
④ Apex-to-stance & Stance-to-apex

Flight



$$\ddot{x} = 0$$
$$\ddot{y} = -g$$

⑤ Stance phase



$$T = \frac{1}{2} m v^2 = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$$

$$V = mgy + \frac{1}{2} k (l - l_0)^2$$

$$l = \sqrt{(x - x_c)^2 + y^2}$$

$$V = mgy + \frac{1}{2}\left(\sqrt{(x-x_c)^2 + y^2} - l_0\right)^2$$

$$\mathcal{L} = T - V$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$$

$$Q_j = 0 \qquad q_j = \{x, y\}$$

EOM

$$m\ddot{x} = k(l_0 - l)\frac{(x - x_c)}{l}$$

$$m\ddot{y} = k(l_0 - l)\left(\frac{y}{l}\right) - mg$$

$$l = \sqrt{(x - x_c)^2 + y^2}$$