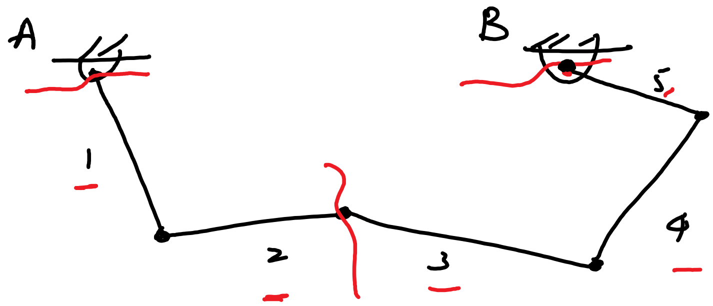
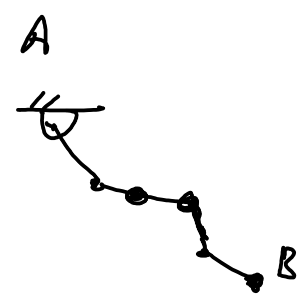
Modelling, simulation & animation of closed-loop systems.



1) Break the system   Break at B.



2) Derive equations of 5-link pendulum

   Euler-Lagrange (E-L)

3) Enforce the closed-loop at B using jacobian.

   Use Q_j on the right side of E-L to enforce constraints at B.

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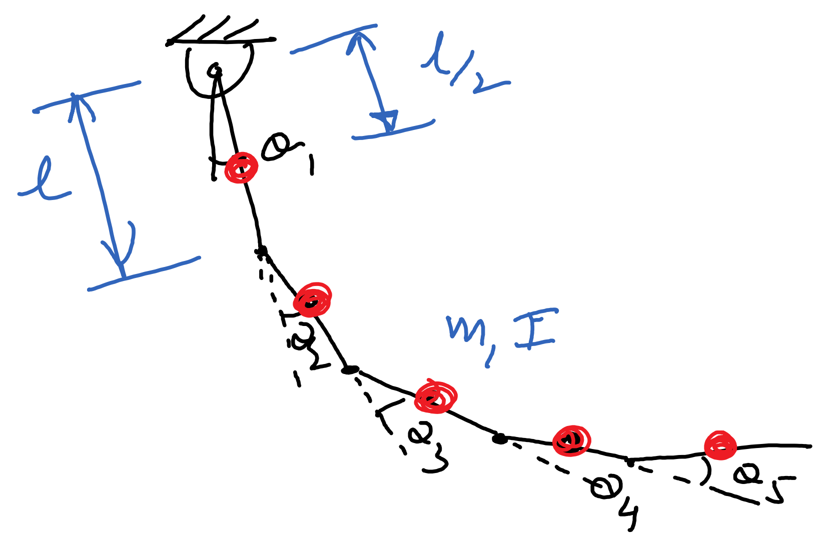
2)



Euler-Lagrange

1) Find positions & velocity

- look at double pendulum

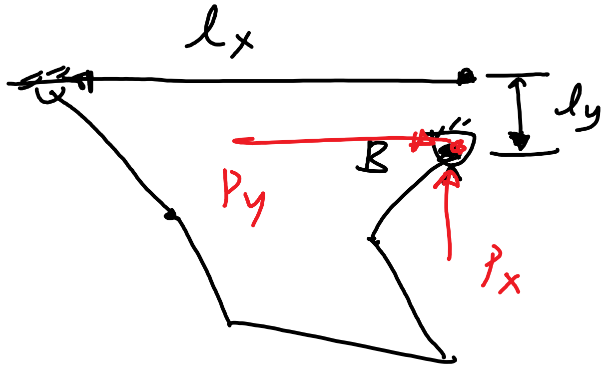
2) $L = T - V$

3) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$

$q_j = \theta_1, \theta_2, \theta_3, \theta_4, \theta_5 \implies 5$ equations

$Q_j = 0$ (for pendulum that has no external forces)

Next, head to MATLAB



$B(x_B, y_B)$

$P_x, P_y = ?$

$\underline{\underline{a_B = 0}} = \ddot{x}_B$

$\ddot{y}_B = 0$

Closed-loop.

(3) $\dfrac{d}{dt}\left(\dfrac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \dfrac{\partial \mathcal{L}}{\partial q_j} = Q_j = 5\times1$ — I

Q_j = torques at $j = 1, 2, 3, 4, 5$

$= J_B^T P = \begin{bmatrix} P_x \\ P_y \end{bmatrix}$

$$J_B = \begin{bmatrix} \partial x_B/\partial \theta_1 & \partial x_B/\partial \theta_2 & \cdots & \partial x_B/\partial \theta_5 \\ \partial y_B/\partial \theta_1 & \partial y_B/\partial \theta_2 & \cdots & \partial y_B/\partial \theta_5 \end{bmatrix}$$

$\underline{\underline{2\times5}}$

$= \underbrace{\underset{5\times2}{J_B^T}\ \underset{2\times1}{P}}_{5\times1} =$

$$M(\theta)\underbrace{\ddot{\theta}}_{5\times 1 \,=\, ?} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \cancel{\tau^{motor}}^{\,0} + \underbrace{J_B^T P}_{2\times 1 \,=\, ?}$$

5 equations

7 unknowns

We are missing 2 constraint $\ddot{x}_B = \ddot{y}_B = 0$

$$\begin{bmatrix} \dot{x}_B \\ \dot{y}_B \end{bmatrix} = \underbrace{\underset{2\times 5}{J_B}\;\underset{5\times 1}{\dot{\theta}}}_{2\times 1} \Rightarrow \dot{z}_B = J_B \dot{\theta}$$

$$0 = \ddot{z}_B = \begin{bmatrix} \ddot{x}_B \\ \ddot{y}_B \end{bmatrix} = J_B \ddot{\theta} + \underset{2\times 5}{\dot{J}_B}\,\dot{\theta}$$ — 2 more equation

Summary

$$M\ddot{\theta} + C\dot{\theta} + G - J_B^T P = 0$$

$$J_B \ddot{\theta} + \dot{J}_B \dot{\theta} = 0$$

$$\Rightarrow \begin{bmatrix} M & -J_B^T \\ J_B & 0 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ P \end{bmatrix} = \begin{bmatrix} -C\dot{\theta} - G \\ -\dot{J}_B \dot{\theta} \end{bmatrix}$$

Pendulum

$$M\ddot{\theta} = -C\dot{\theta} - G$$

Chain

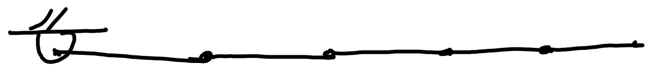
$$\begin{bmatrix} M & -J_B^T \\ J_B & 0 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ p \end{bmatrix} = \begin{bmatrix} -C\dot{\theta} - G \\ -\dot{J}_B \dot{\theta} \end{bmatrix}$$

forces at B

$$\begin{array}{c} 5\times 1 \\ 2\times 1 \end{array} \begin{bmatrix} \ddot{\theta} \\ p \end{bmatrix} = \begin{bmatrix} M & -J_B^T \\ J_B & 0 \end{bmatrix}^{-1} \begin{bmatrix} -C\dot{\theta} - G \\ -\dot{J}_B \dot{\theta} \end{bmatrix}$$

Simulations

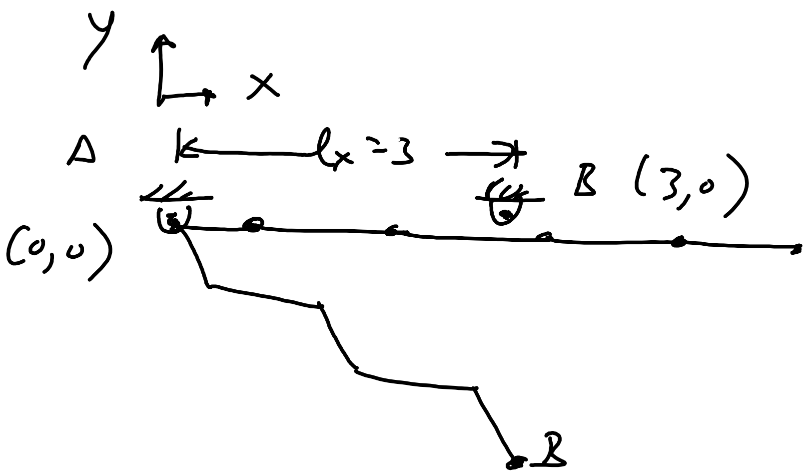
Pendulums



$\theta_1 = 90^\circ$ $\quad \theta_2 = \theta_3 = \theta_4 = \theta_5 = 0$

$\dot\theta_1 = \dot\theta_2 = \dot\theta_3 = \dot\theta_4 = \dot\theta_5 = 0$ ✓

Chains



Need to ensure $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ are such that

① $\left.\begin{array}{l} x_B = l_x = 3 \\ y_B = l_y = 0 \end{array}\right\}$

→ Inverse kinematics to get

↳ Use fsolve

2 output

5 inputs

$\boxed{\theta_1, \ldots \theta_5}$

② $\left.\begin{array}{l} \dot x_B = 0 \\ \dot y_B = 0 \end{array}\right\}$

Find $\boxed{\dot\theta_1, \dot\theta_2, \dot\theta_3, \dot\theta_4, \dot\theta_5}$ such that

fsolve — 2 outputs

5 inputs $\dot\theta_1, \dot\theta_2, \ldots \dot\theta_5$